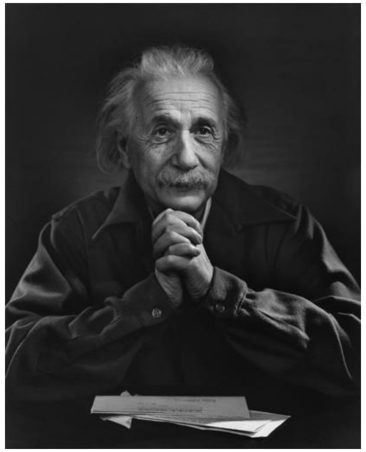
Supermassive Black Hole Mergers

by Vikram Manikantan

Astro 300A: Dynamics, Spring 2025
University of Arizona

Supermassive Black Hole Mergers

Qualitatively self-explanatory.
Quantitatively : $\mathrm{M_{BH}} > 10^5 M_{\odot}$

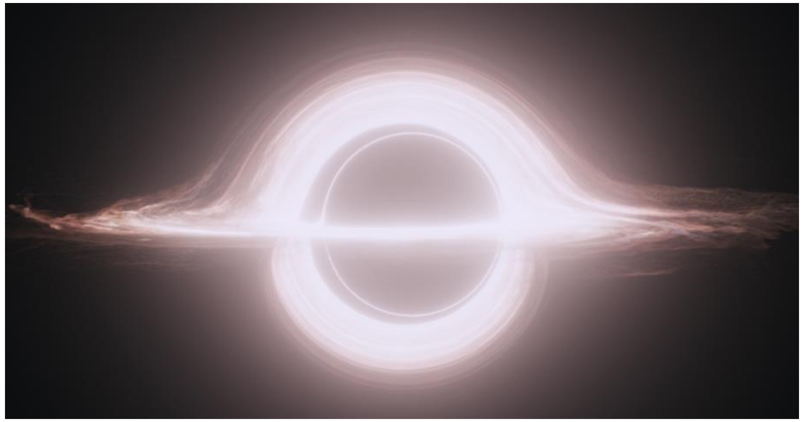


Albert Einstein, credits: Yousuf Karsh

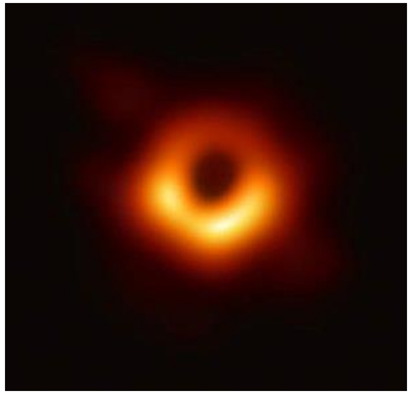
"a combination of two things, ~~especially companies,~~ into one"
- Oxford Dictionary

Black Holes – what are they?

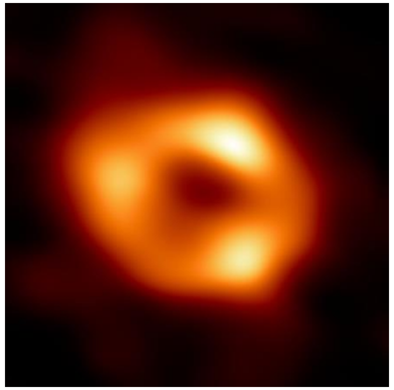
- A region of spacetime separated by an event horizon
- What is an event horizon?
- Point at which no particle (not even photons) can escape to infinity
- Spacetime and Geometry – Sean Carroll



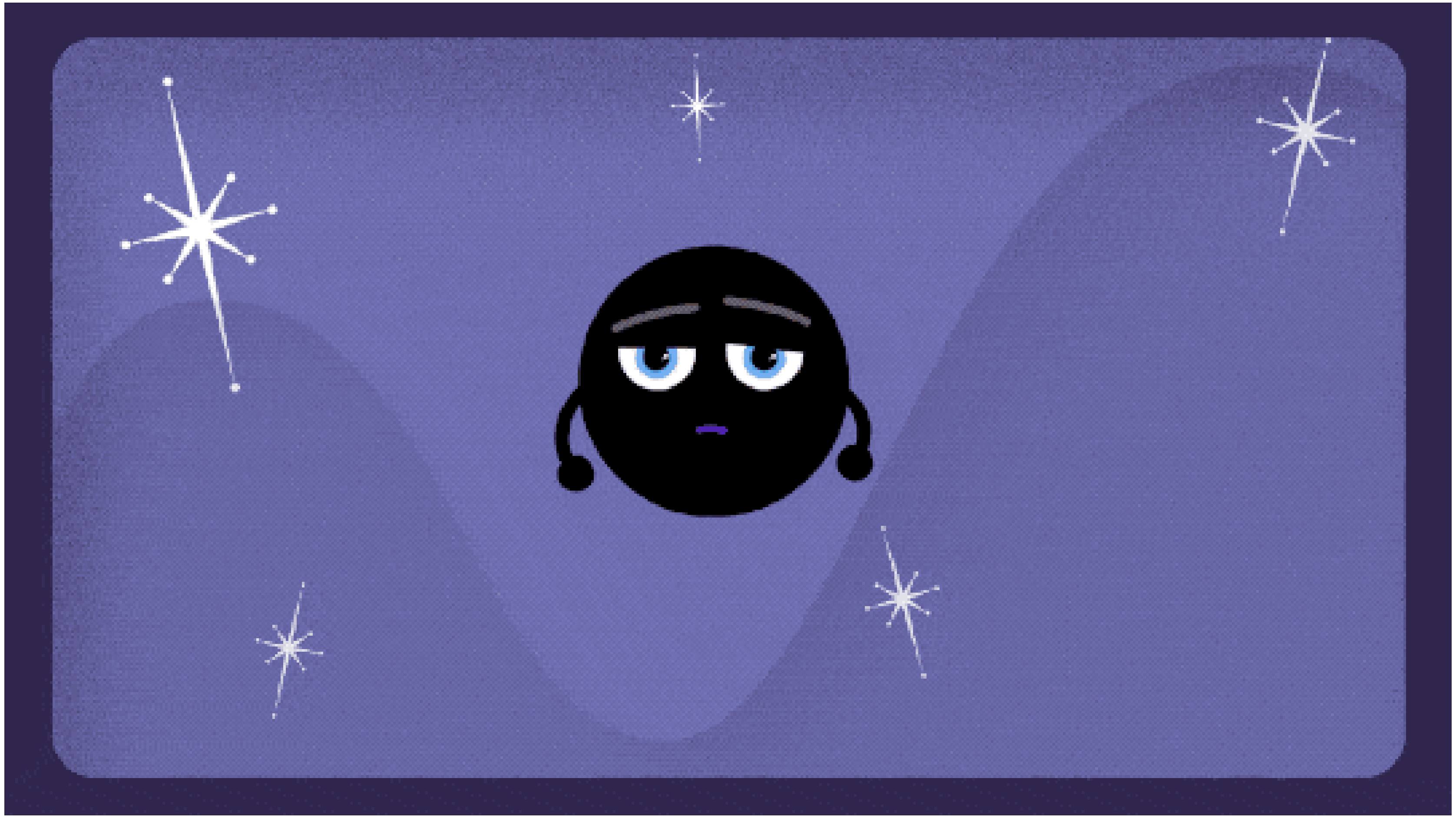
Interstellar, 2014



M87*, EHT

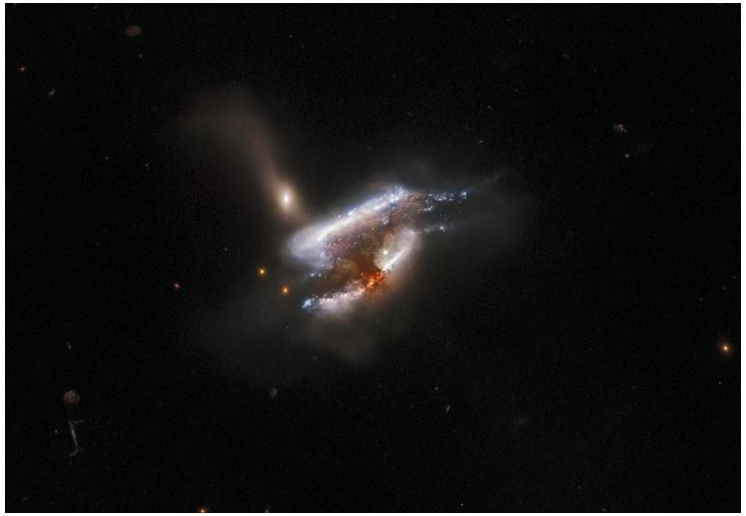


Sag A*, EHT


Credit: NASA's Goddard Space Flight Center

SMBH Mergers: what do we know?

1. Galaxies exist
2. They host central SMBHs (usually)
3. They merge often

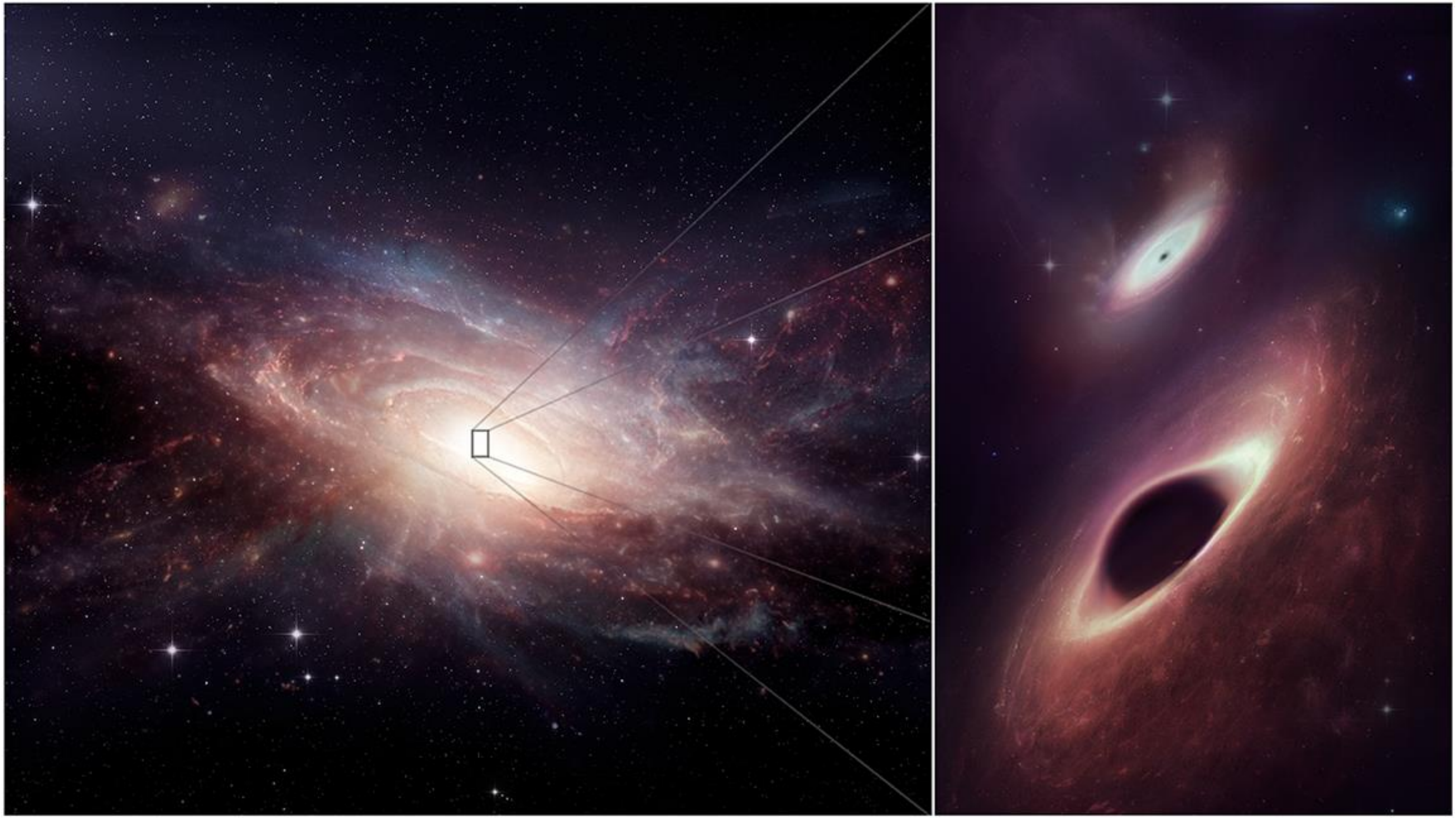


What we don't know

1. Galaxies exist
2. They host central SMBHs (usually)
3. They merge often

1. What happens to the central black holes? (BHs)

2. How do they merge? And what does it look like?



https://www.scientificamerican.com/article/colliding-supermassive-black-holes-discovered-in-nearby-galaxy/

Concepts we are going to use:

1. Order-of-mag estimates
2. Orbital dynamics (and decay)
3. Dynamical Friction
4. Gravitational waves

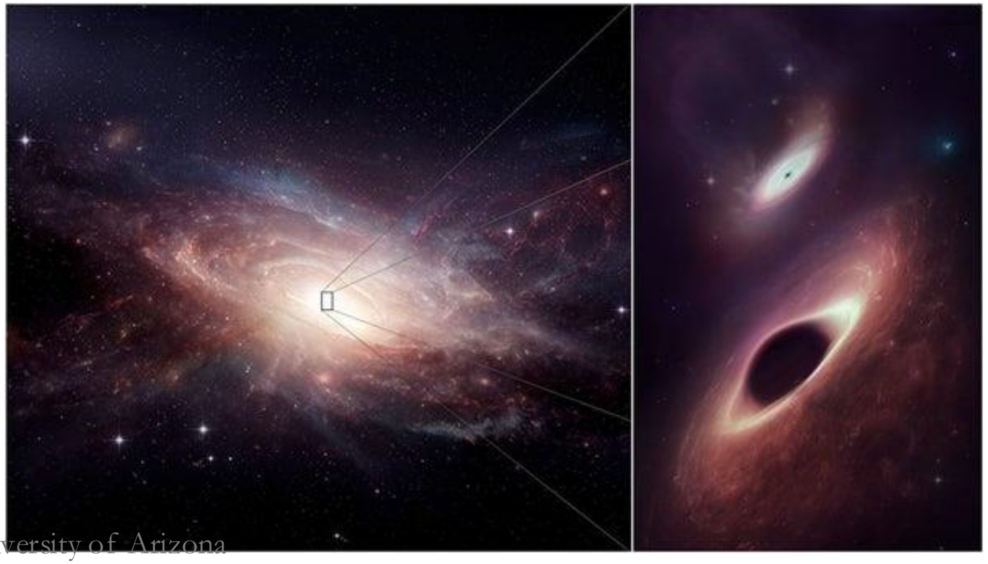
After this lecture, you will be able to:

1. **Give context** to the importance of SMBHs
2. **Explain** the key processes that lead to SMBH merger
3. **Use** order-of-mag estimates to determine SMBH merger timescales

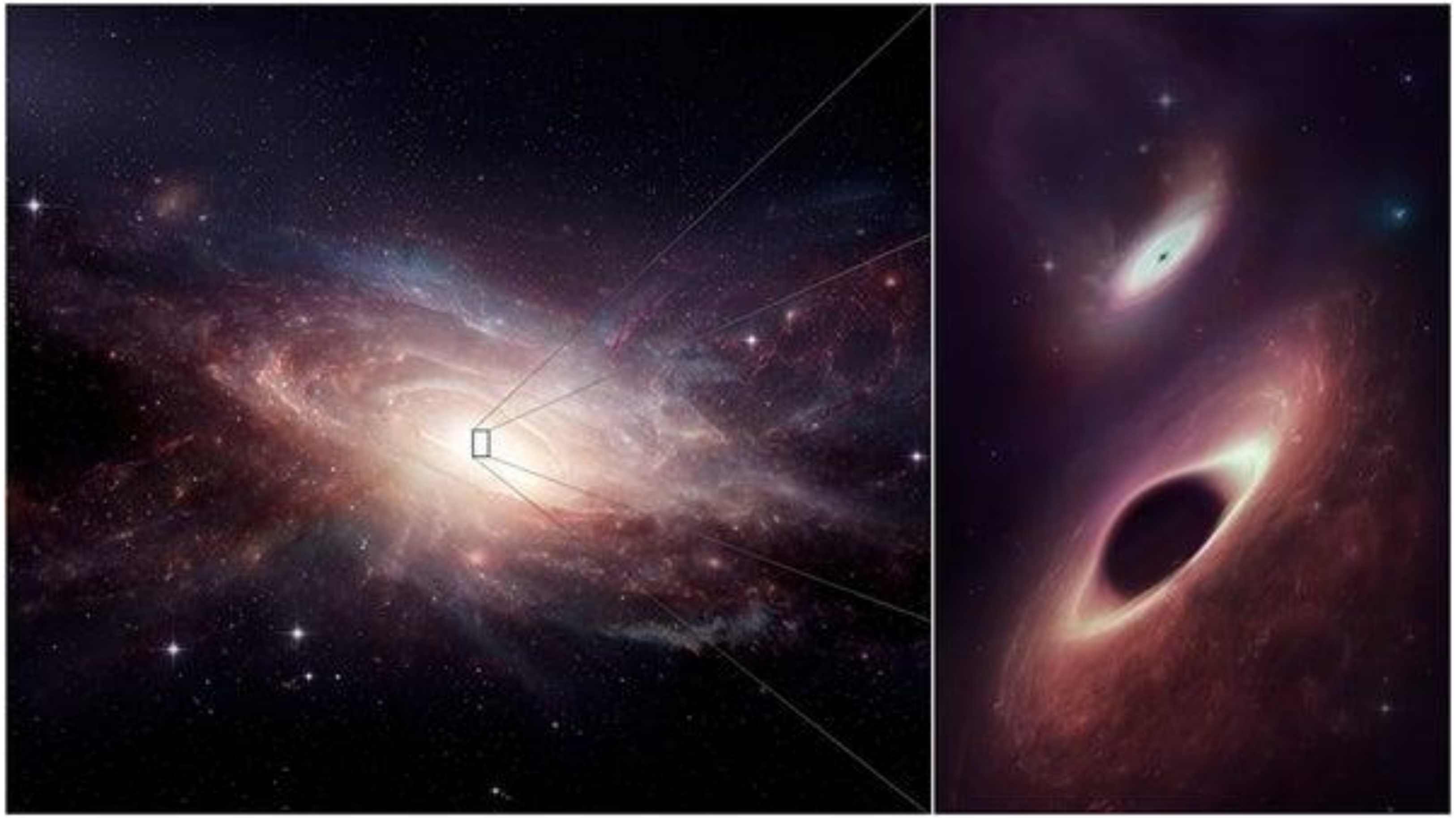
* There is an in-class assignment today

Why do we care about SMBHs merging?

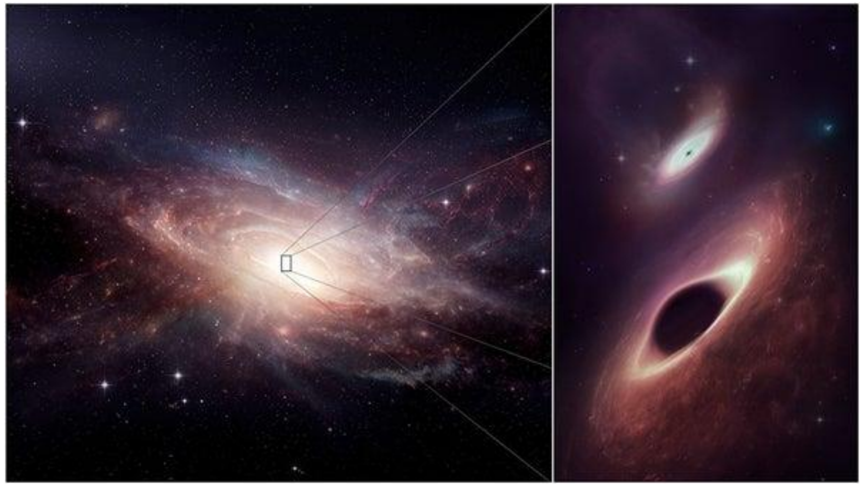
- Black holes are cool
- Where SMBHs come from are a huge open question in astrophysics?
- Trace galaxy mergers throughout cosmic time
- Einstein's theory of GR is great, but *is it correct*?
- What about the behavior of particles in relativistic jets?

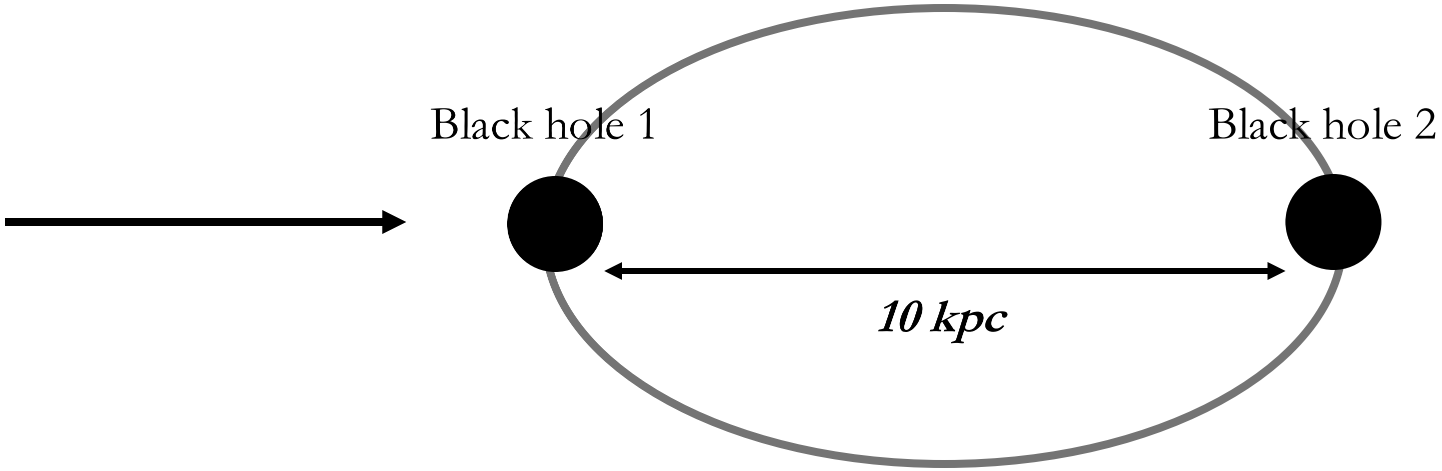


Simplify a complex problem



First, orbiting supermassive black holes

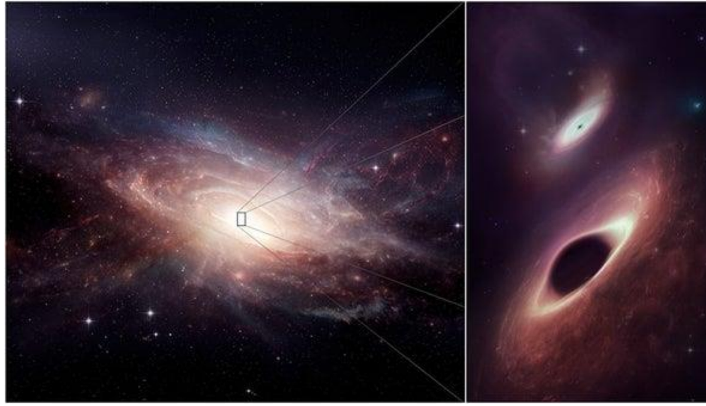


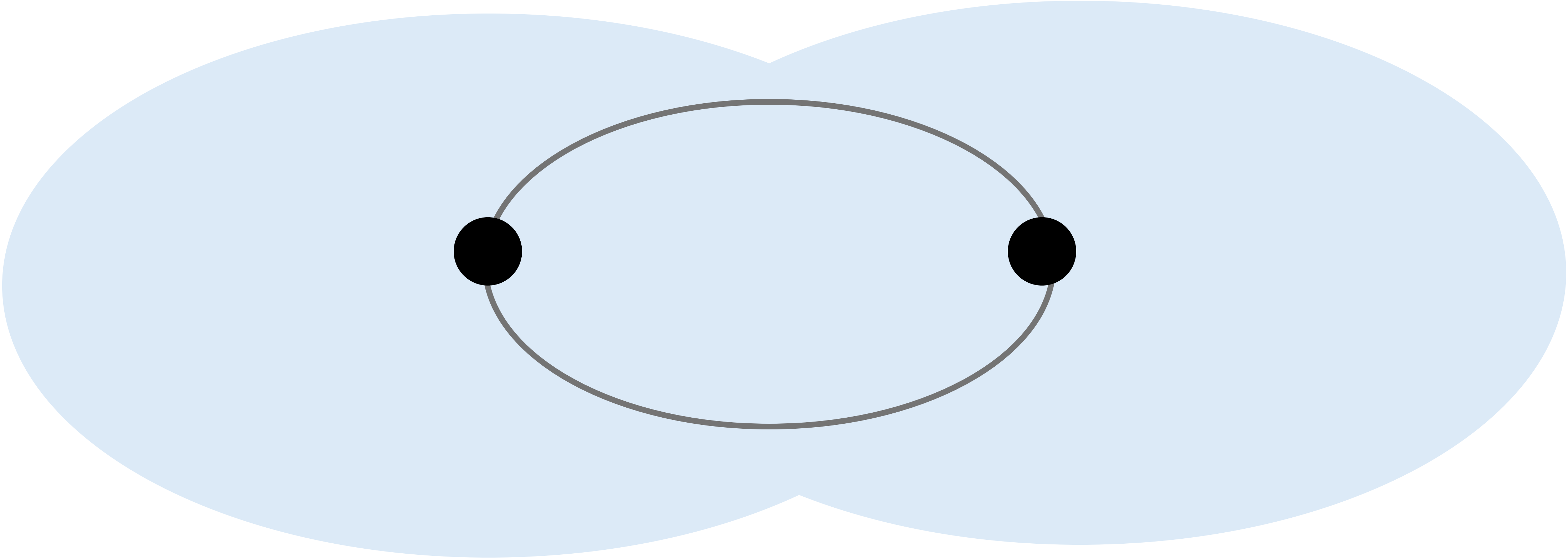


Not to scale

What else is there?

Interstellar Gas





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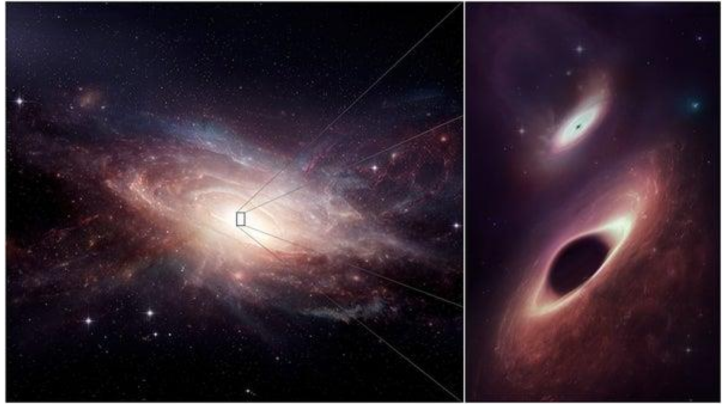
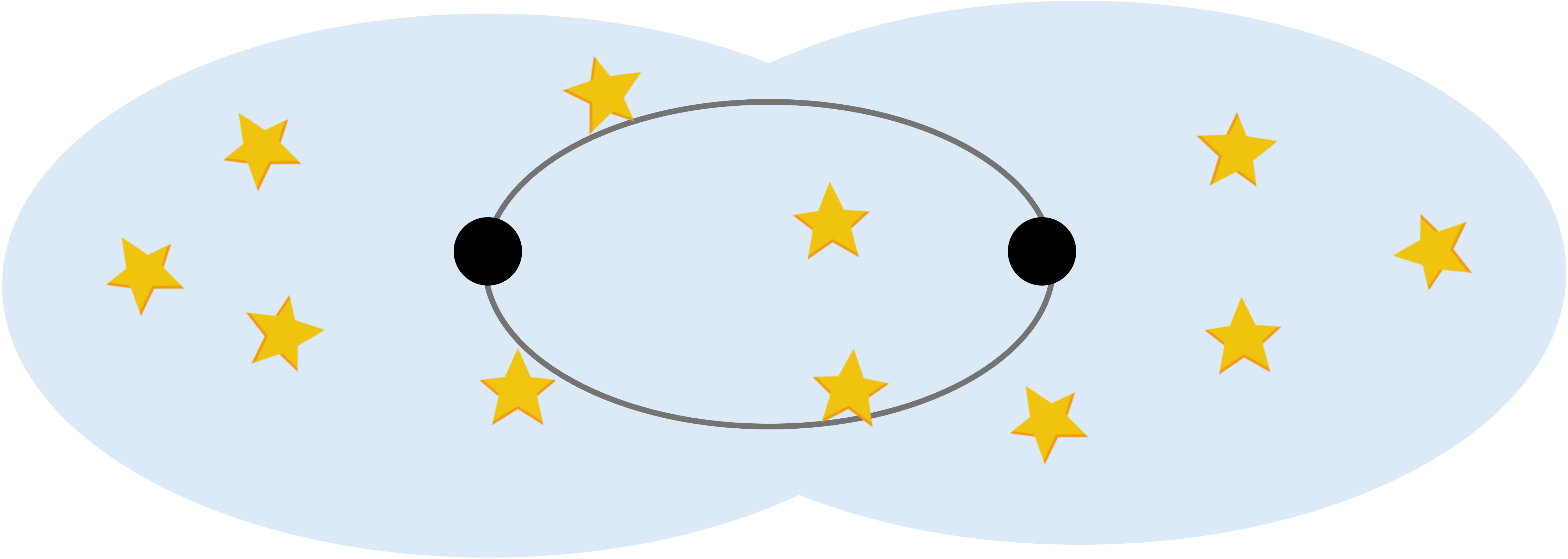
What else is there?

Stars





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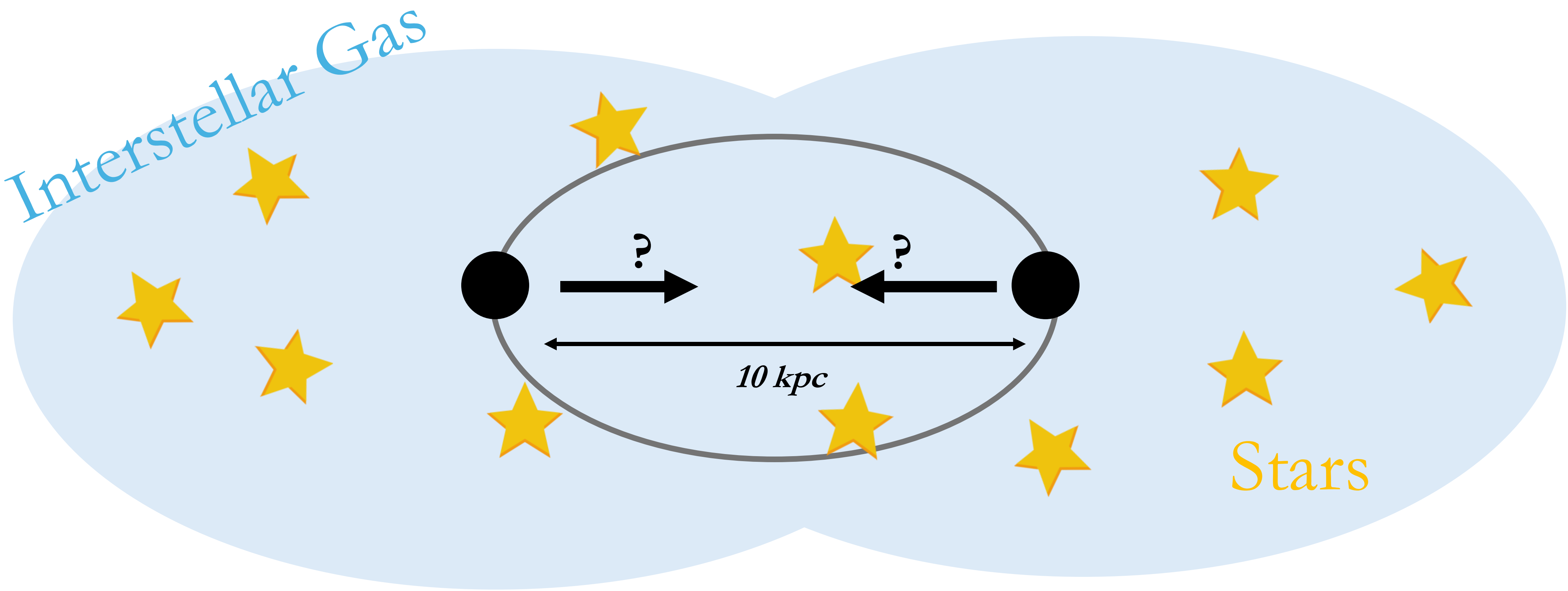
What needs to happen for these BHs to merge?



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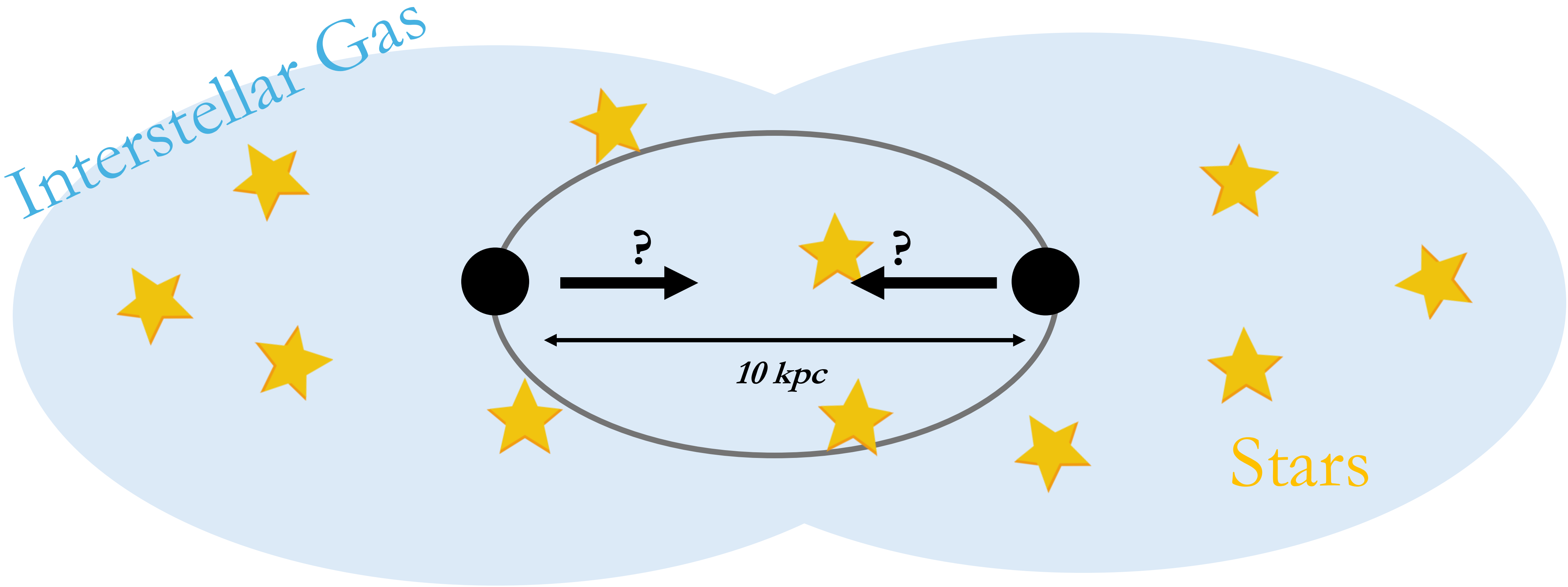
We want to get rid of angular momentum! How?



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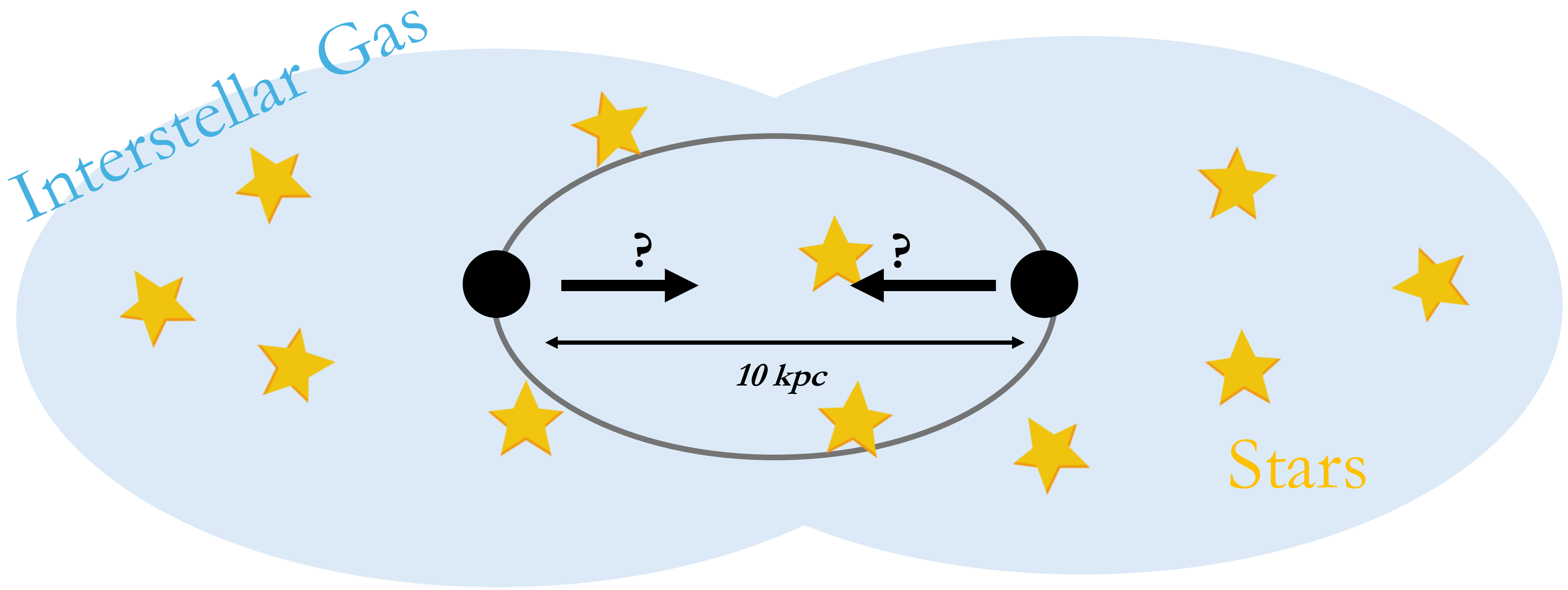

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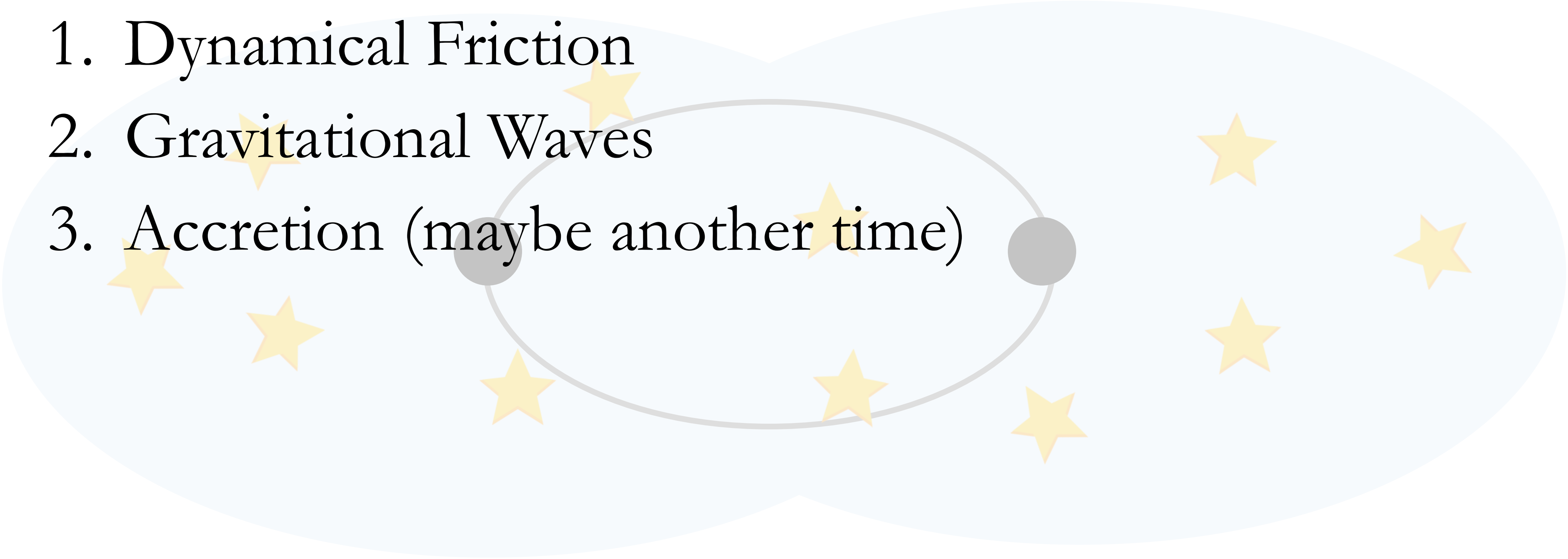
Angular Momentum Extraction

1. Dynamical Friction
2. Gravitational Waves
3. Accretion (maybe another time)



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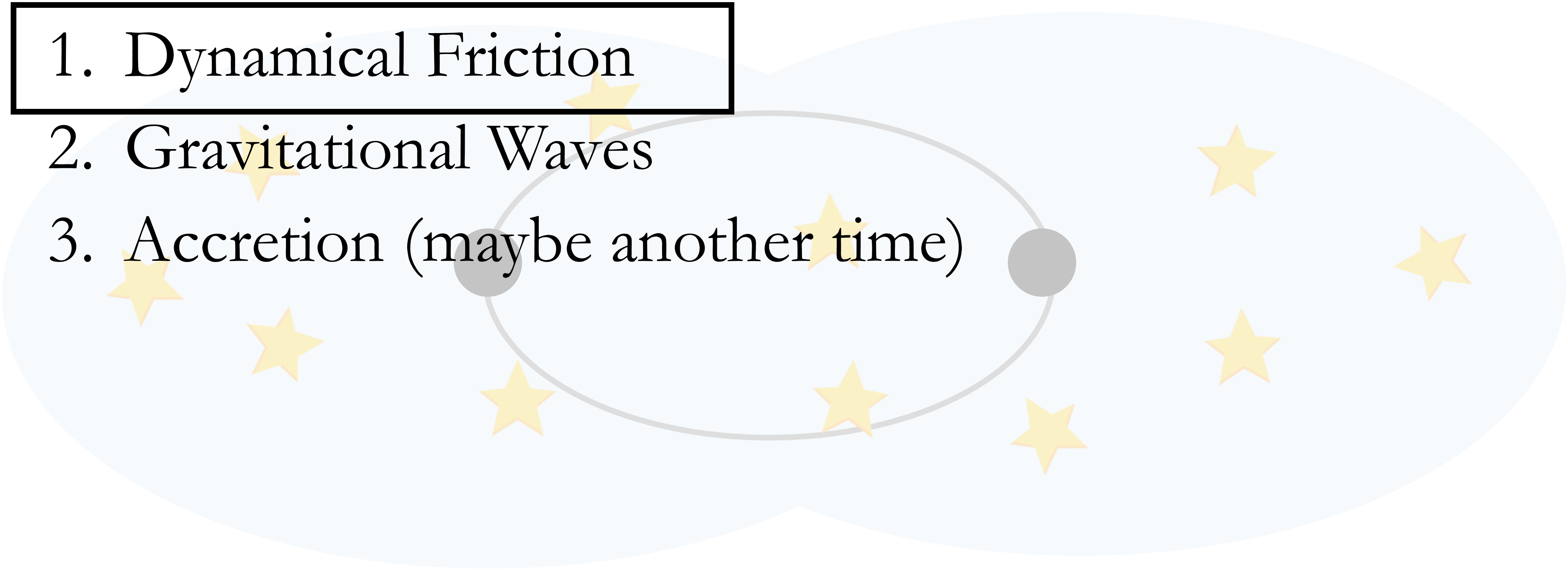

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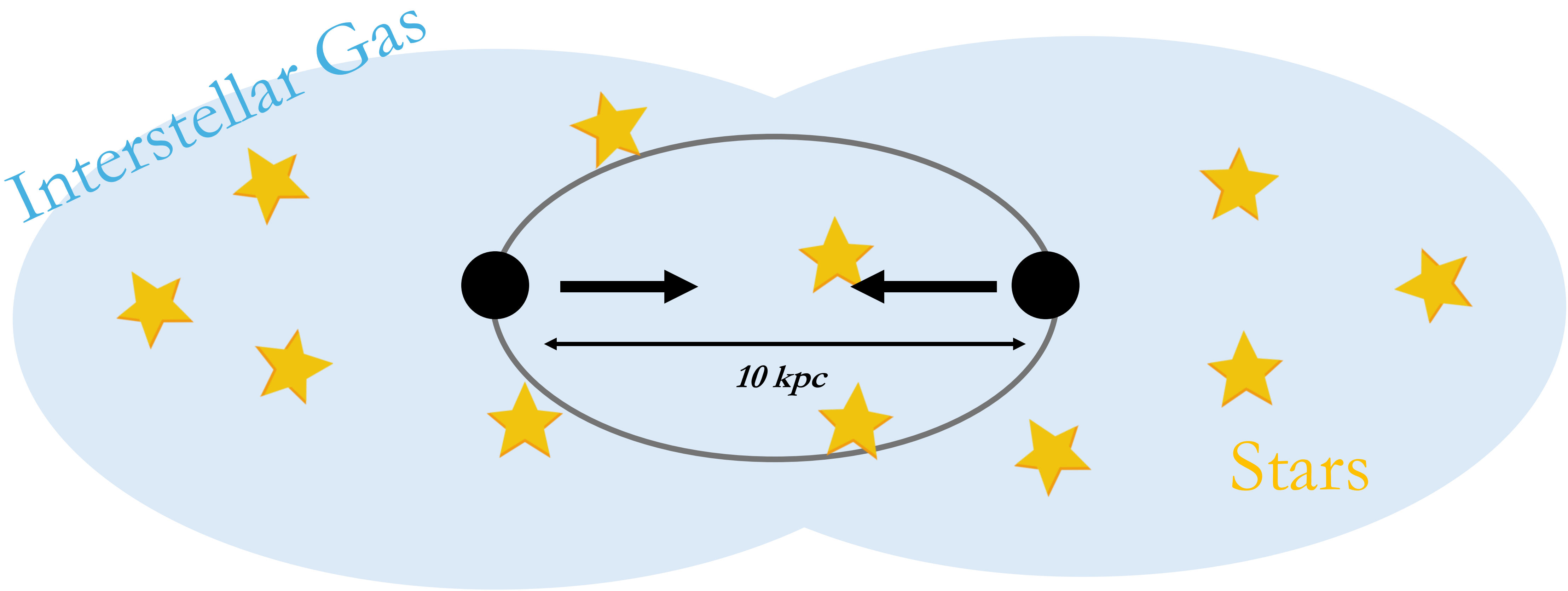
Dynamical friction through interactions with stars and gas



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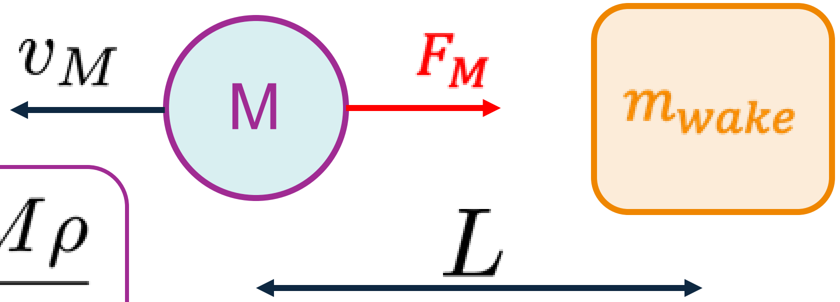
Dynamical Friction



$$\frac{dv_M}{dt} \propto -\frac{G^2 M \rho}{v_M^2}$$

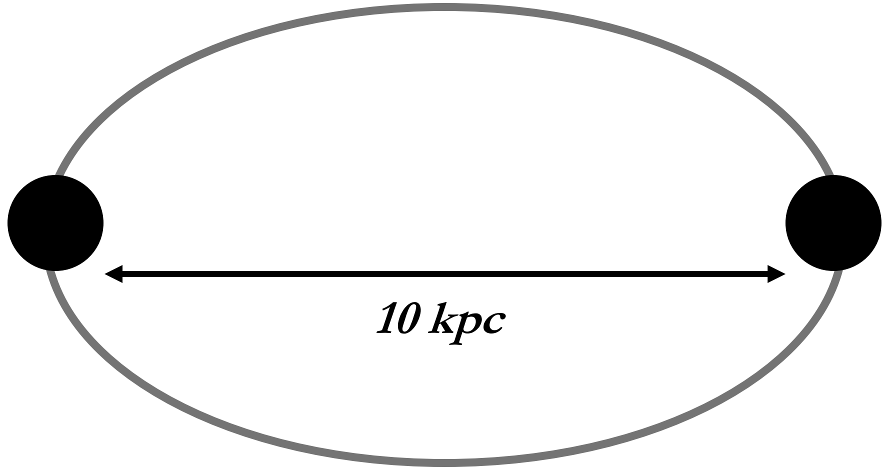
$$\frac{d\vec{v}_M}{dt} = -\frac{4\pi \ln(\Lambda) G^2 \rho M}{v_M^2} \hat{v}_M$$

How do we find the time it takes to slow the BHs down to zero velocity?

From HW6:

$$\Rightarrow T_{stop} = \frac{v_M^3}{12\pi \ln \Lambda \, G^2 \rho M}$$

Dynamical friction



$$\Rightarrow T_{stop} = \frac{v_M^3}{12\pi \ln \Lambda \, G^2 \rho M}$$

- Assuming the total binary mass is ***M = 10^7 Msun***
- and they have orbital separation ***a = 10 kpc*,**
- Using Kepler's laws, their orbital velocities are, ***v ~ 2 km/s***
- Calculate how long it takes to bring the BH to a stop?

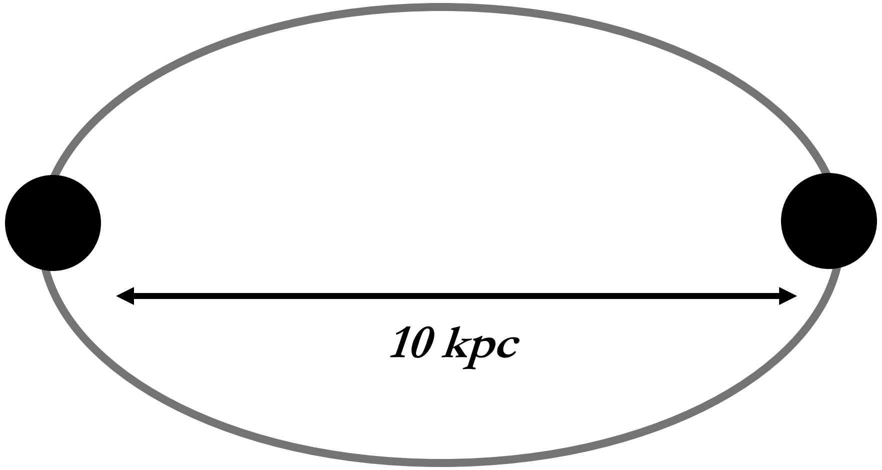
Dynamical friction

$$\Rightarrow T_{stop} = \frac{v_M^3}{12\pi(10)G^2\rho M}$$

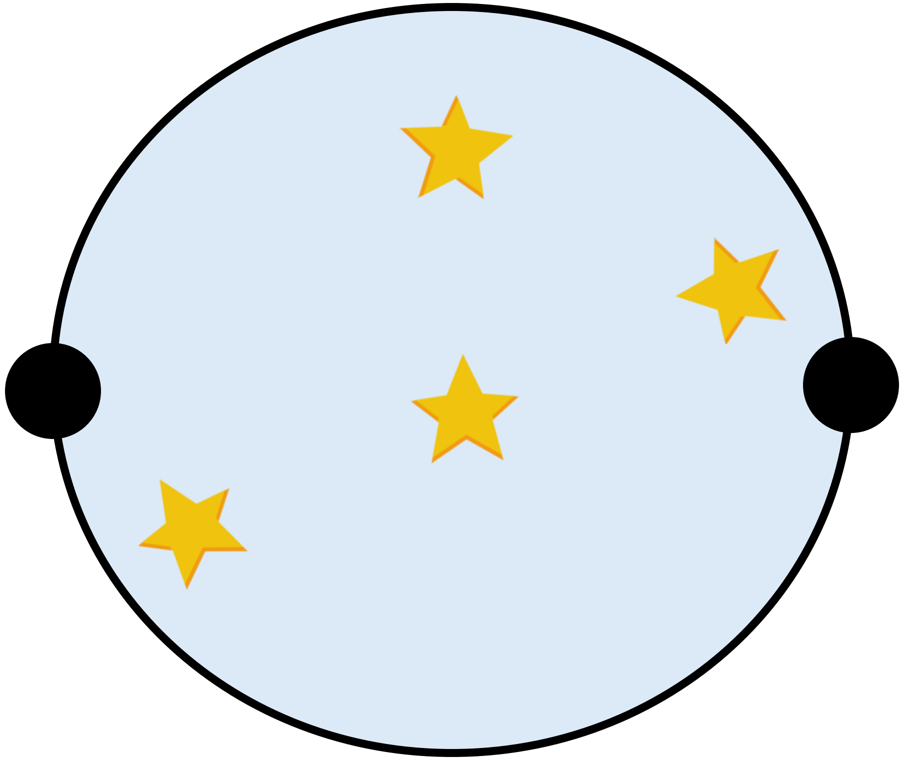
$$\frac{T_{stop}}{950\ years} = \left(\frac{v_M}{200km\ s^{-1}}\right)^3$$

$$\Rightarrow T_{stop} \sim 0.00095\ years \sim 8.4\ hours$$

- Assuming the same parameters as in the homework
- We don't have to do all the work again



This is an incomplete picture!

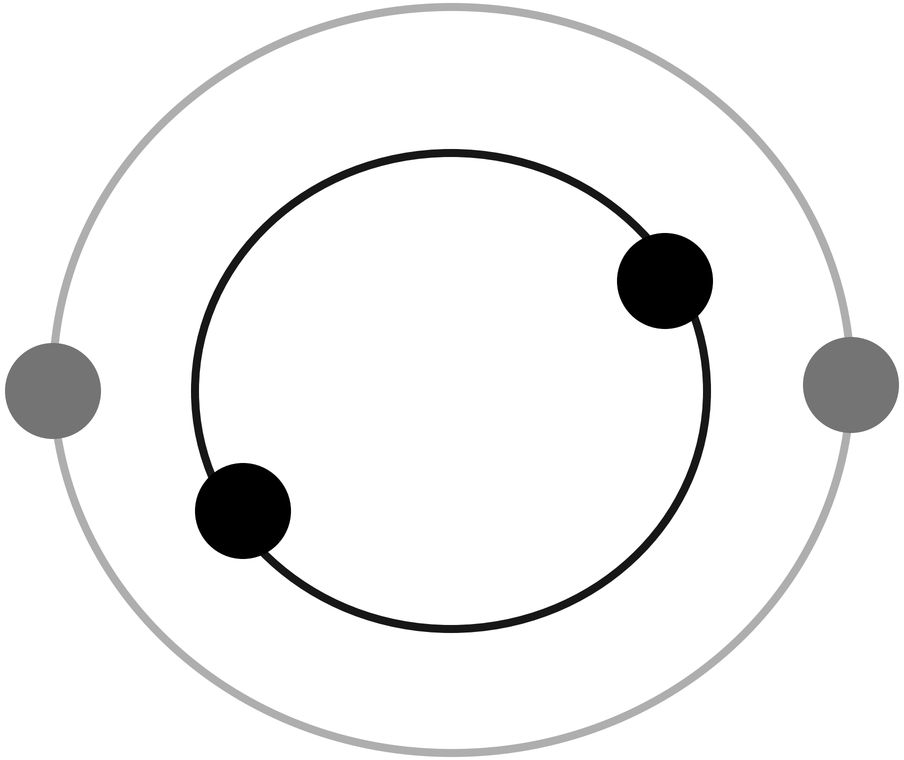


- The mass of this system is not just the BH mass!
- Remember enclosed mass from HW7?
- The gravitational mass is really the mass of the galaxies:

$$M_{MW} = 10^{12}\ Msun$$

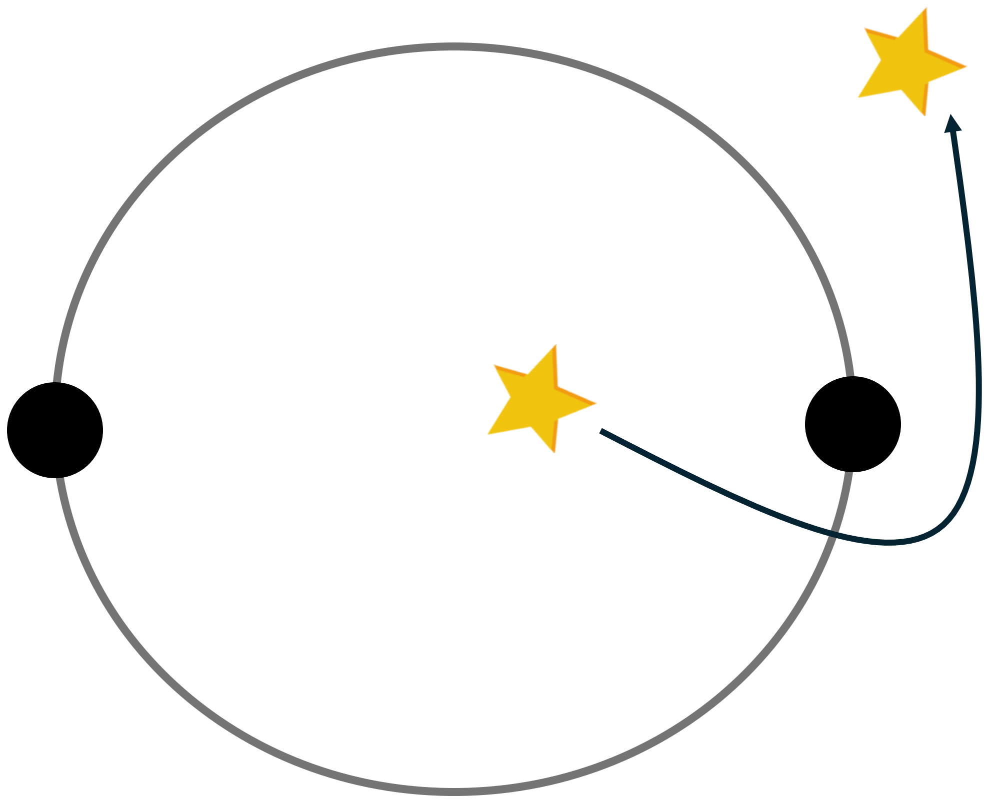
- Recalculating the Keplerian velocity: ***v = 600 km/s***
- **T_{stop} ~ 27 * 950 yrs ~ 25,000 yrs**

This is an incomplete picture!



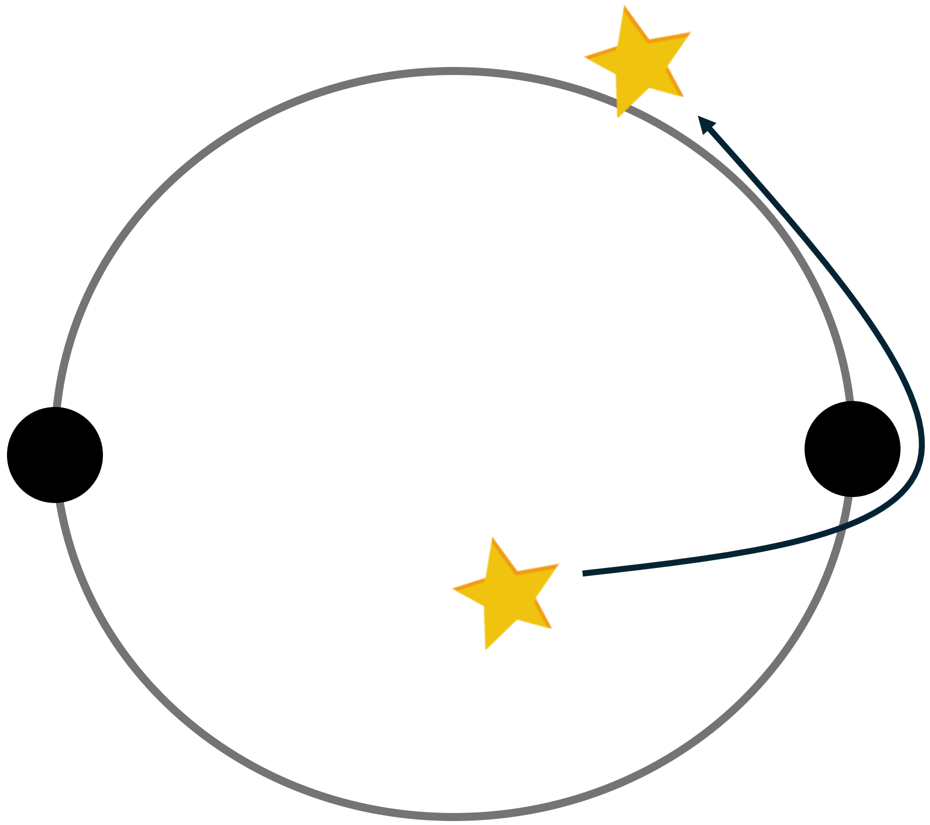
- Slowing down leads to a new orbital separation, not to merger!
- The BHs will speed up again, according to Kepler's law
- So, we need to think of a more complete method of angular momentum loss

Angular Momentum loss from stellar interactions



- Stars with low angular momentum (closer to the SMBH binary) exist
- This is called the 'loss cone'
- They interact with a BH and get ejected, extracting angular momentum
- If done efficiently, this can bring the SMBHs closer to each other

How would you approach solving this?

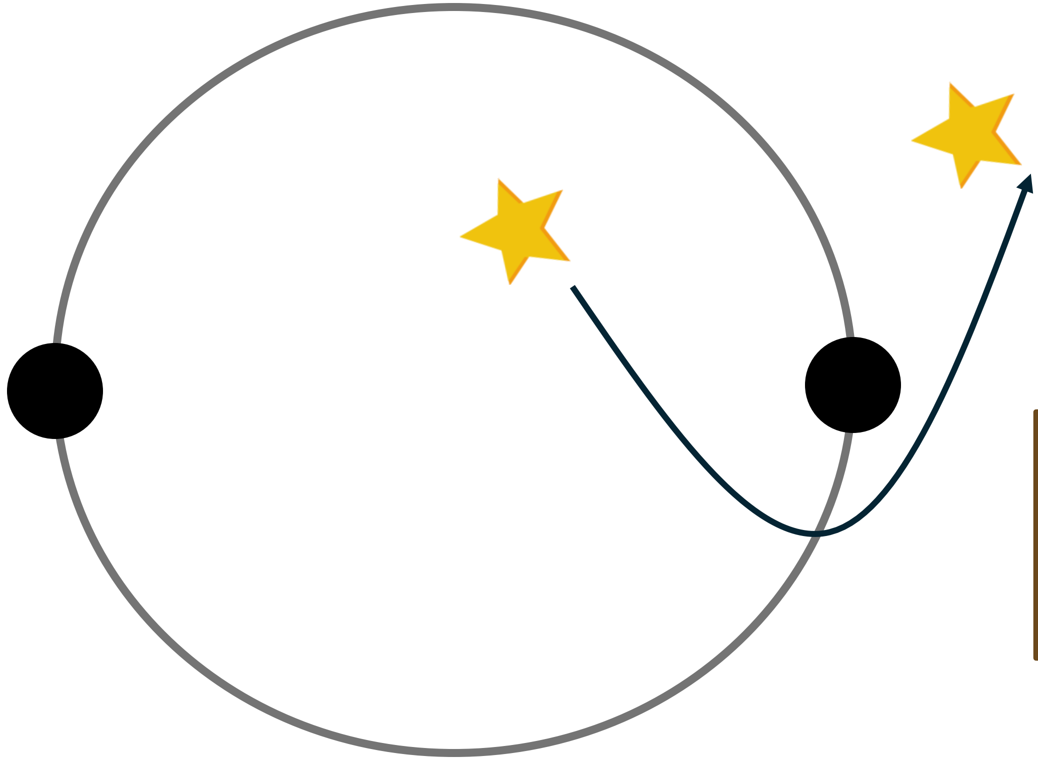


- Instead of velocity, let's think about angular momentum

$$\frac{d\vec{v}_M}{dt} = -\frac{4\pi \ln(\Lambda) G^2 \rho M}{v_M^2} \hat{v}_M$$

$$\Rightarrow T_{stop} = \frac{v_M^3}{12\pi \ln \Lambda \, G^2 \rho M}$$

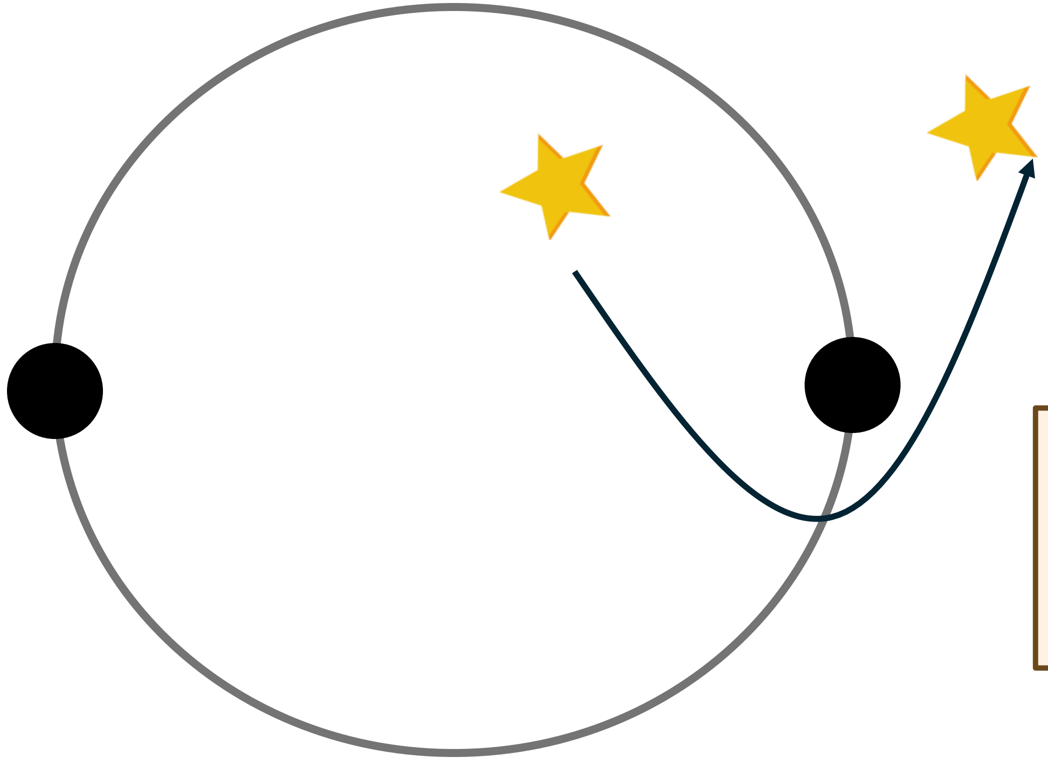
How do we quantify angular momentum loss?



- Define a total **L**
- Define a **dL/dt** using our best friend, dimensional analysis

$$\frac{dL}{dt} = ??$$

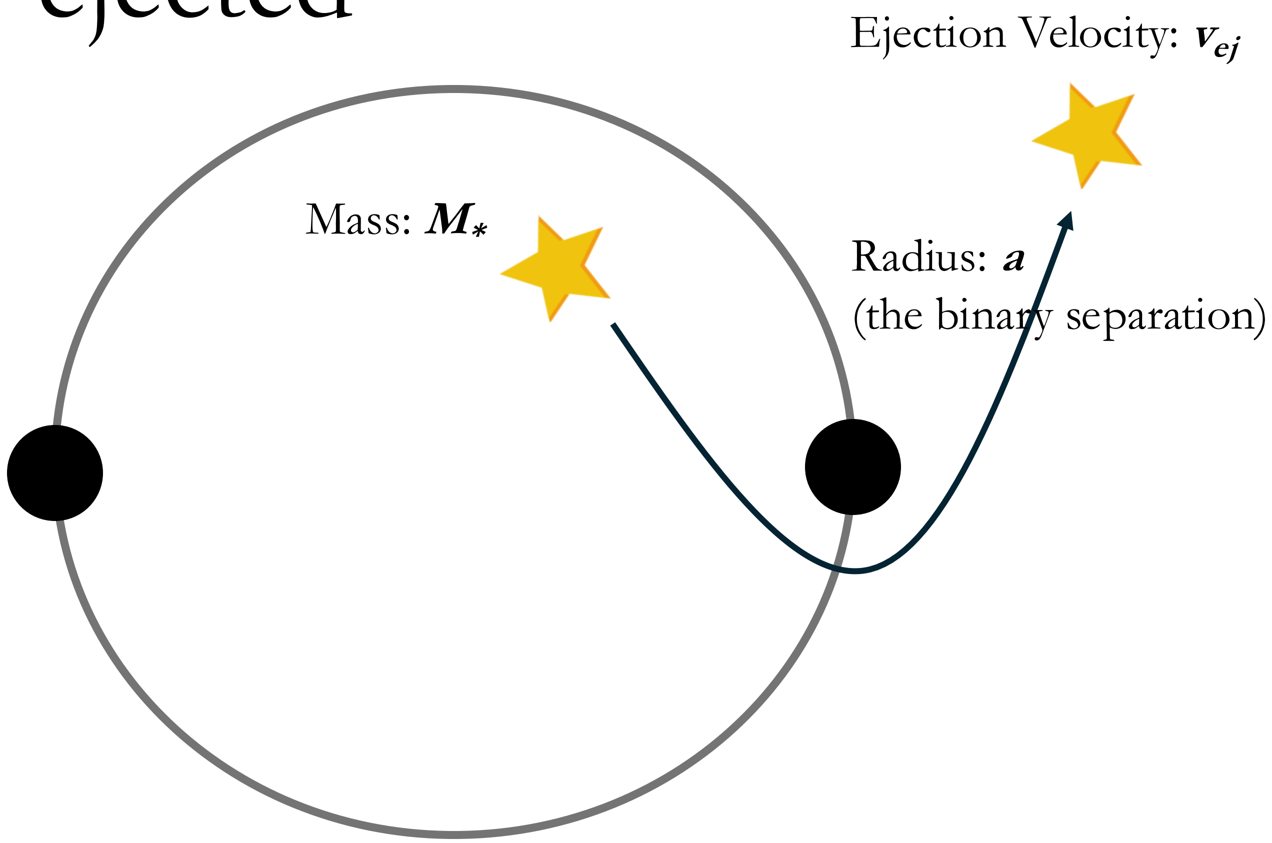
How do we quantify angular momentum loss?



- Define a total **L**
- Define a **dL/dt** using our best friend, dimensional analysis

$$\frac{dL}{dt} = \Delta L/\Delta t$$

Assume a star interacts with the binary, and is ejected



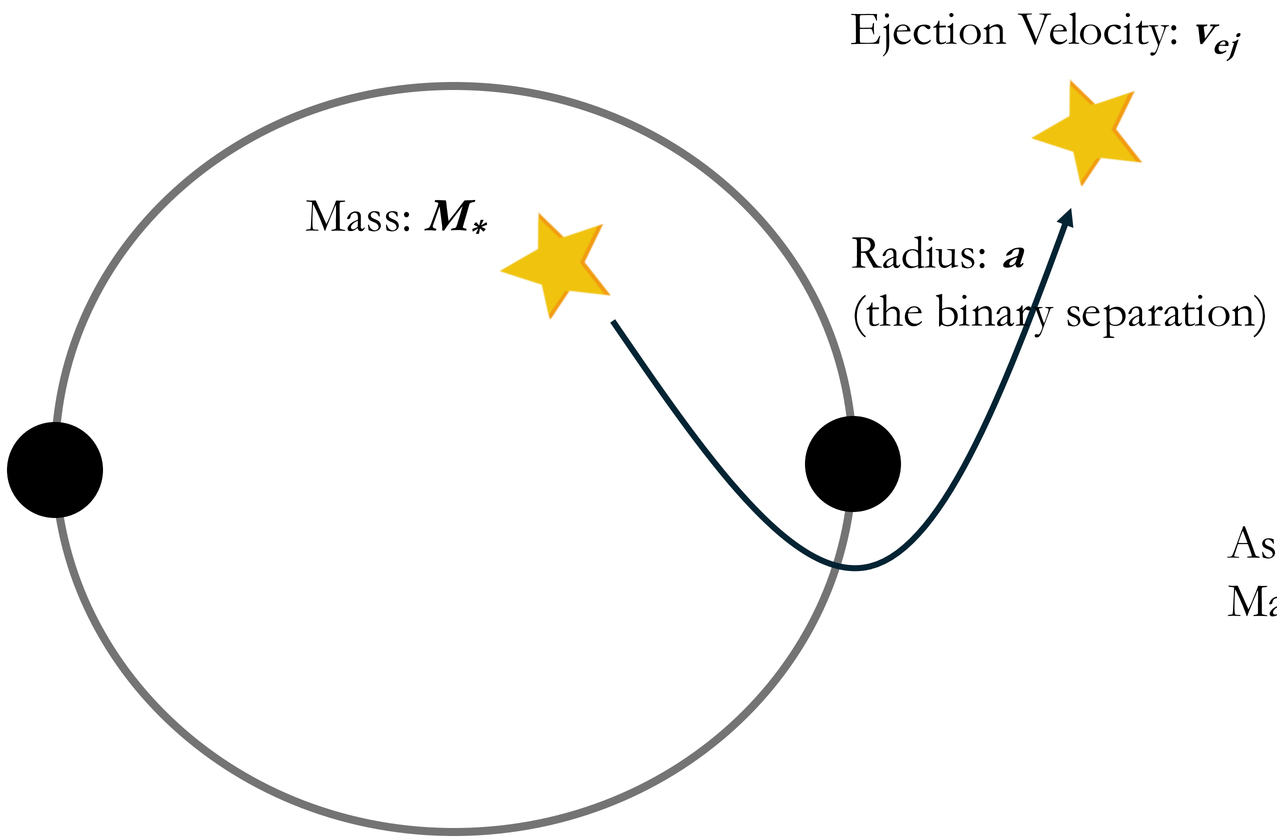
$$\frac{dL}{dt} = \frac{\Delta L}{\Delta t}$$

$$\Delta L = ?$$

$$\Delta L = M_* v_{ej} a$$

REMEMBER: this is for **one** stellar interaction

Generalize to a rate of stellar interactions



$$\frac{dL}{dt} \sim \frac{\Delta L}{\Delta t}$$

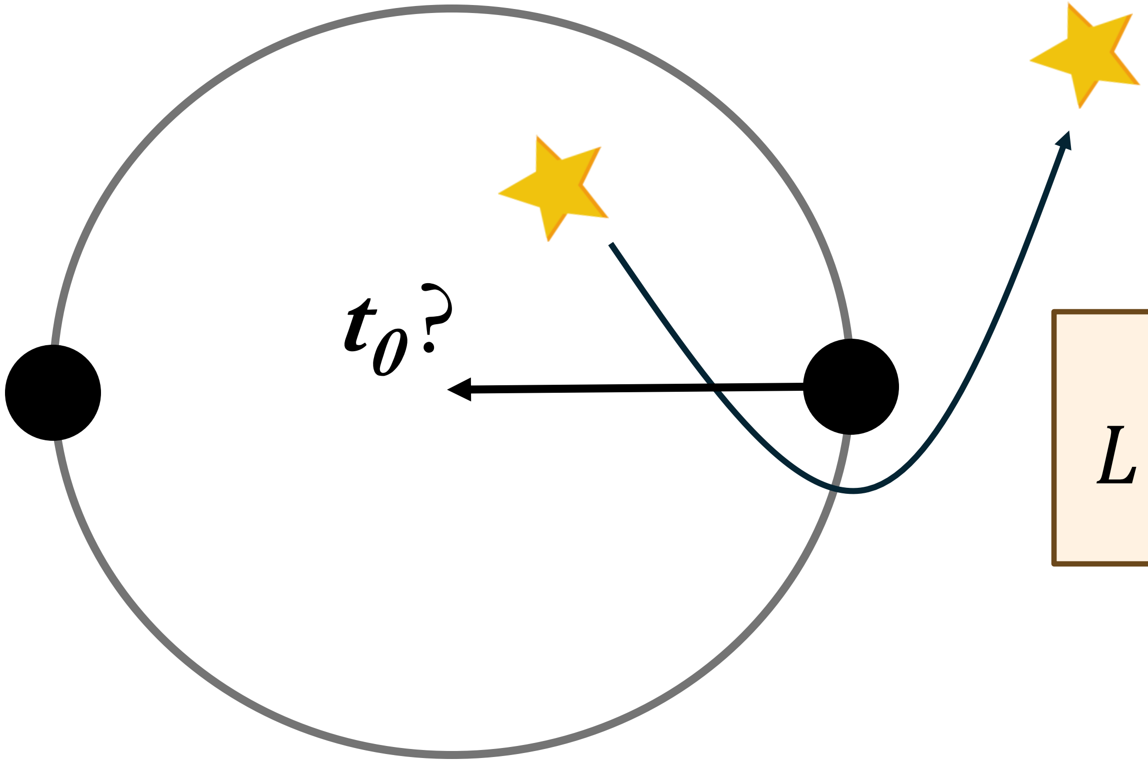
$$\Delta L = M_* v_{ej} a$$

Assume n_{int} stellar interactions per year, and let's make the Mass and velocity averages, such that:

$$\frac{dL}{dt} = \bar{M}_* \bar{v}_{ej} a \, n_{int}$$

The units check out!

Now, find time it takes to get ***L*** to ***0***



$$\frac{dL}{dt} = \bar{M}_* \bar{v}_{ej} a \, n_{int}$$

Not a function of ***L***…

$$L = M\sqrt{GMa} \Rightarrow a = \frac{L^2}{GM^3}$$

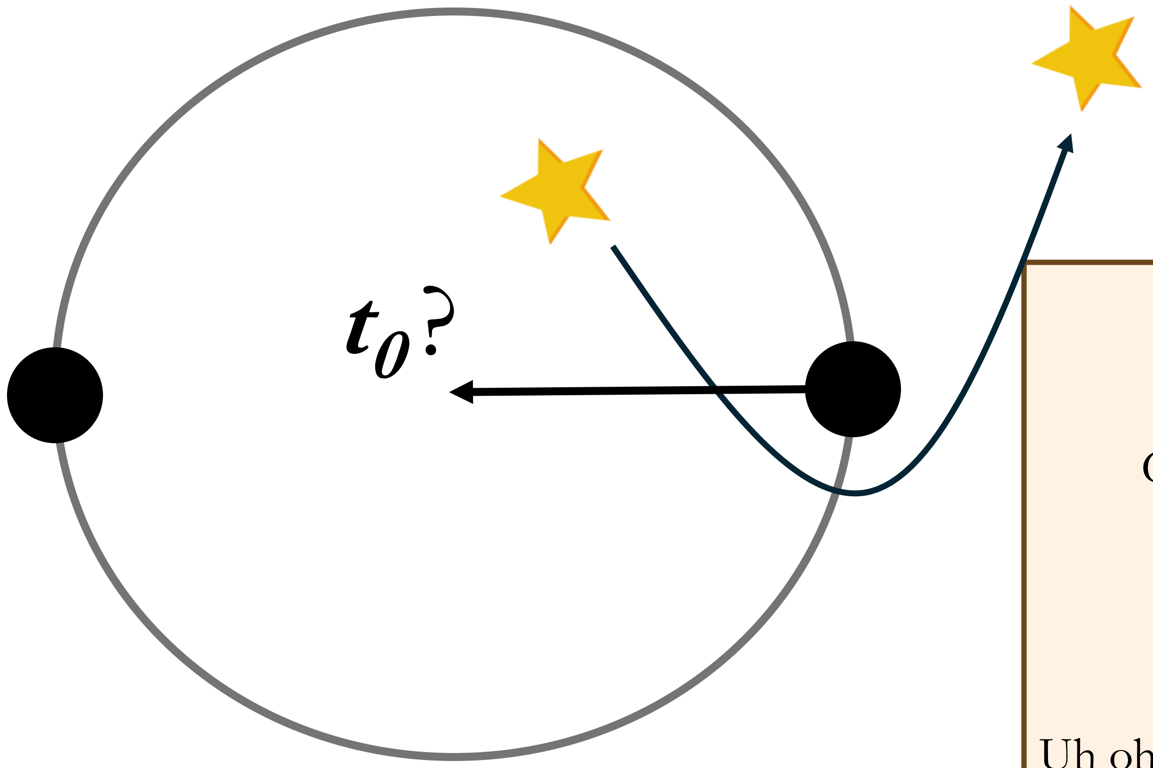
$$t_0 = ?$$

Recall from HW6:

$$\frac{d\vec{v}_M}{dt} = -\frac{4\pi \ln(\Lambda) G^2 \rho M}{v_M^2} \hat{v}_M$$

$$\Rightarrow T_{stop} = \frac{v_M^3}{12\pi \ln \Lambda \, G^2 \rho M}$$

Now, find time it takes to get ***L*** to some ***0***



$$\frac{dL}{dt} = -\bar{M}_* \bar{v}_{ej} \frac{L^2}{GM^3} n_{int}$$

$$dL = -\bar{M}_* \bar{v}_{ej} \frac{L^2}{GM^3} n_{int} dt$$

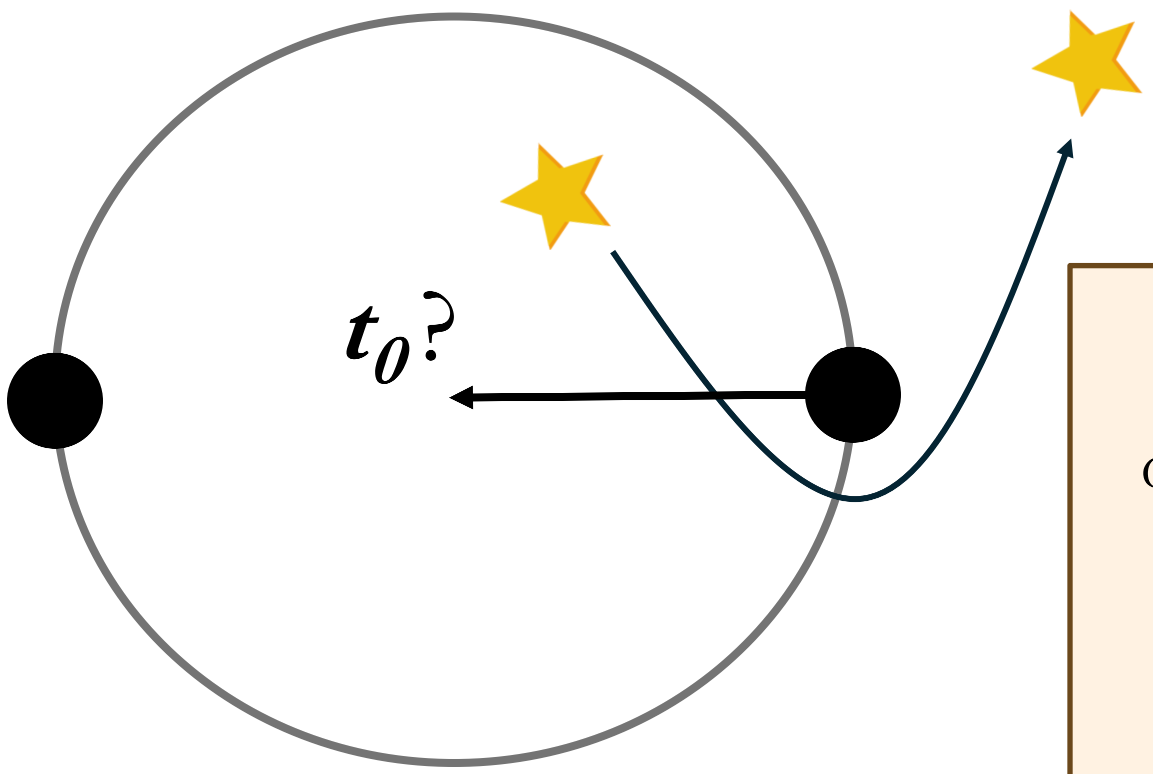
Collect like terms and integrate both sides:

$$\int_{L_0}^{0} \frac{1}{L^2} dL = \int_{0}^{t_0} \frac{\bar{M}_* \bar{v}_{ej} n_{int}}{GM^3} dt$$

Uh oh

$$\left[-\frac{1}{L}\right]_{L_0}^{0} = \frac{\bar{M}_* \bar{v}_{ej} n_{int}}{GM^3} t_0$$

Now, find time it takes to get $\boldsymbol{L}$ to some $\boldsymbol{L_{min}}$



$$\frac{dL}{dt} = -\bar{M}_* \bar{v}_{ej} \frac{L^2}{GM^3} n_{int}$$

$$dL = -\bar{M}_* \bar{v}_{ej} \frac{L^2}{GM^3} n_{int} dt$$

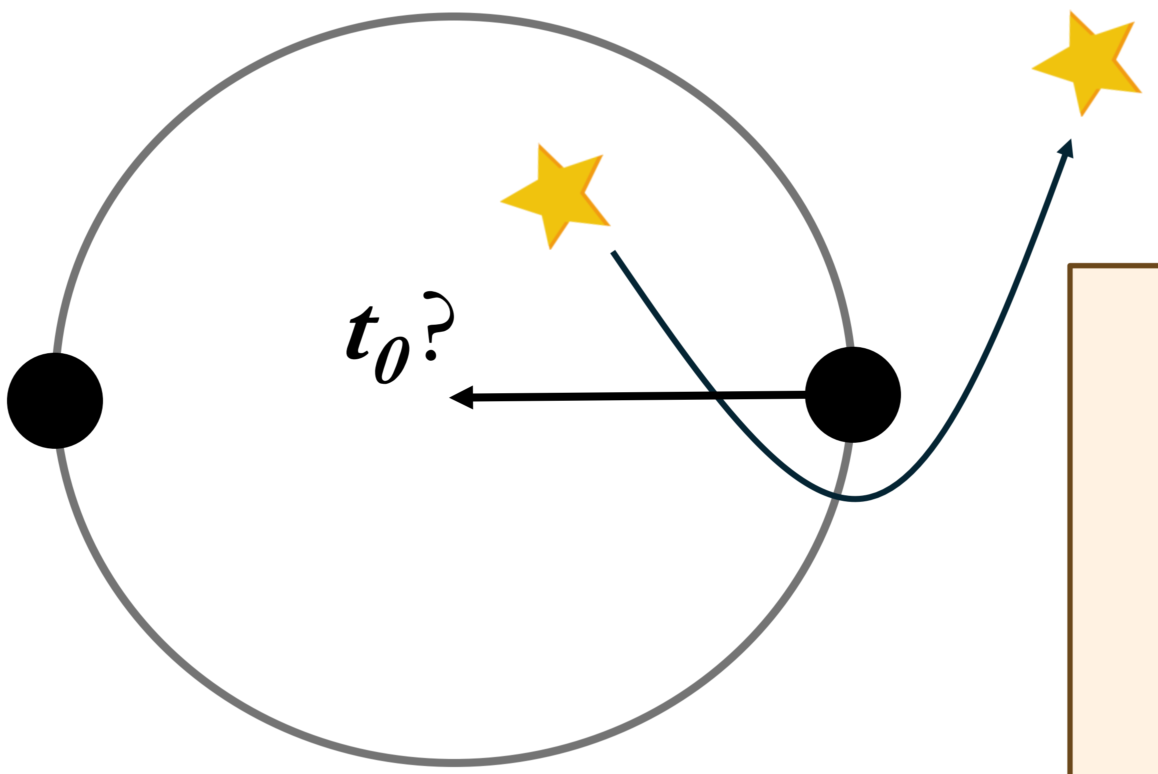
Collect like terms and integrate both sides:

$$\int_{L_0}^{0} \frac{1}{L^2} dL = \int_{0_0}^{t} \frac{\bar{M}_* \bar{v}_{ej} n_{int}}{GM^3} dt$$

$$\left[-\frac{1}{L}\right]_{L_0}^{L_{min}} = -\frac{\bar{M}_* \bar{v}_{ej} n_{int}}{GM^3} t_0$$

$$\left(\frac{1}{L_0} - \frac{1}{L_{min}}\right) = -\frac{\bar{M}_* \bar{v}_{ej} n_{int}}{GM^3} t_0$$

Now, find time it takes to get $\boldsymbol{L}$ to some $\boldsymbol{L_{min}}$



$$\frac{dL}{dt} = -\bar{M}_* \bar{v}_{ej} \frac{L^2}{GM^3} n_{int}$$

Re-arrange for t_0:

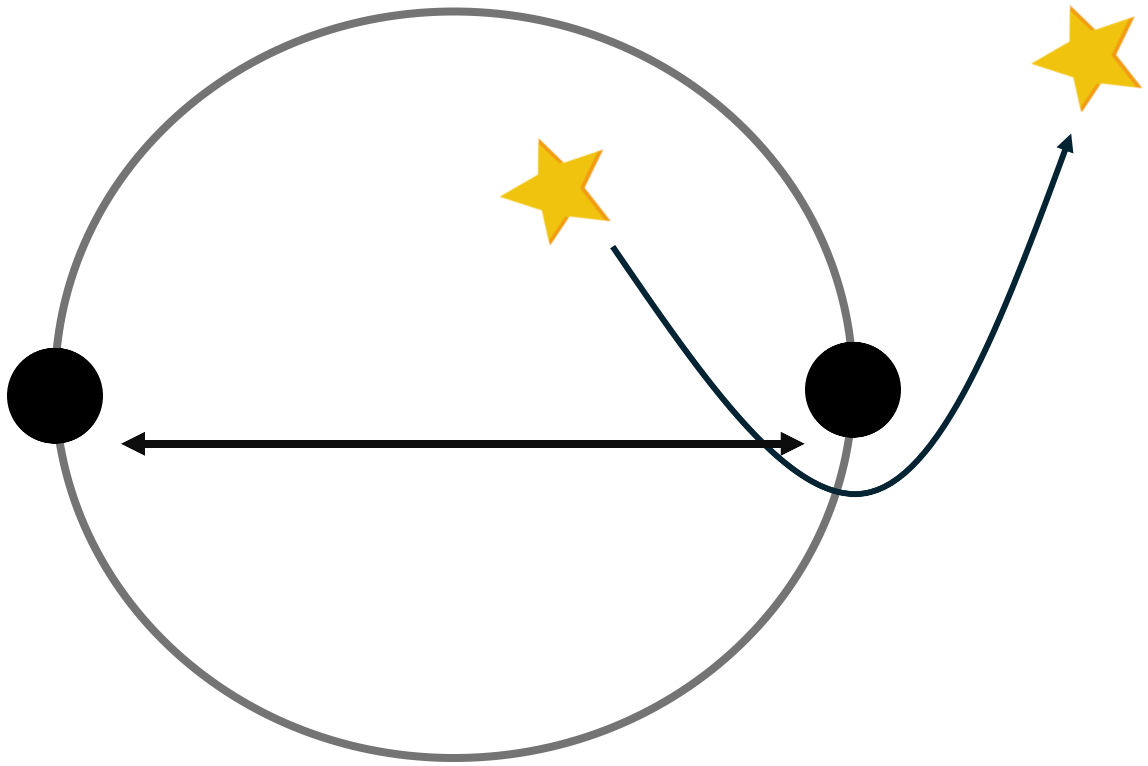
$$\left(\frac{1}{L_0} - \frac{1}{L_{min}}\right) = -\frac{\bar{M}_* \bar{v}_{ej} n_{int}}{GM^3} t_0$$

$$\Rightarrow t_0 = \frac{GM^3}{\bar{M}_* \bar{v}_{ej} n_{int}} \left(\frac{1}{L_{min}} - \frac{1}{L_0}\right)$$

We can make a few more simplifications, like replacing L with known quantities:

$$L = M\sqrt{GMa}$$

We did it! Just with guessing

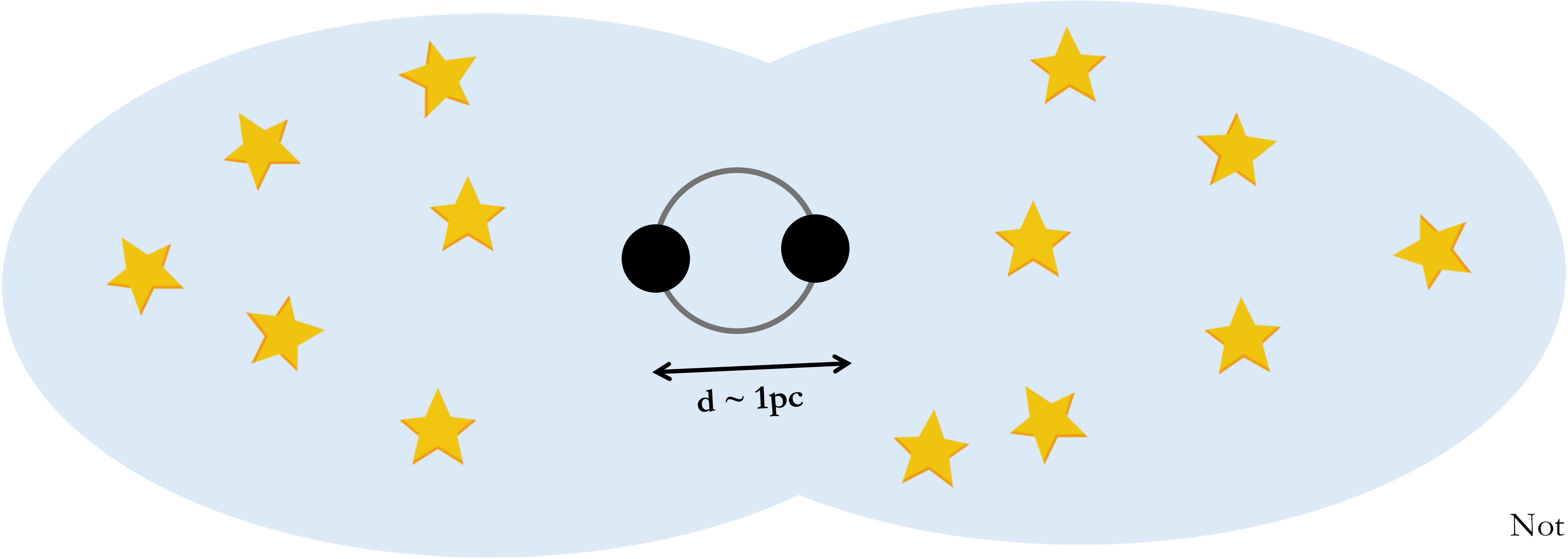


$$\frac{dL}{dt} = -\bar{M}_* \bar{v}_{ej} \frac{L^2}{GM^3} n_{int}$$

$$\Rightarrow t_0 = \frac{\sqrt{GM^3}}{\bar{M}_* \bar{v}_{ej} n_{int}} \left(\frac{1}{a_{min}} - \frac{1}{a_0} \right)$$

- How long is this applicable?
- Recall the stellar density of the milky way
- Turns out: **ϱ ~ 1 star/pc^3**

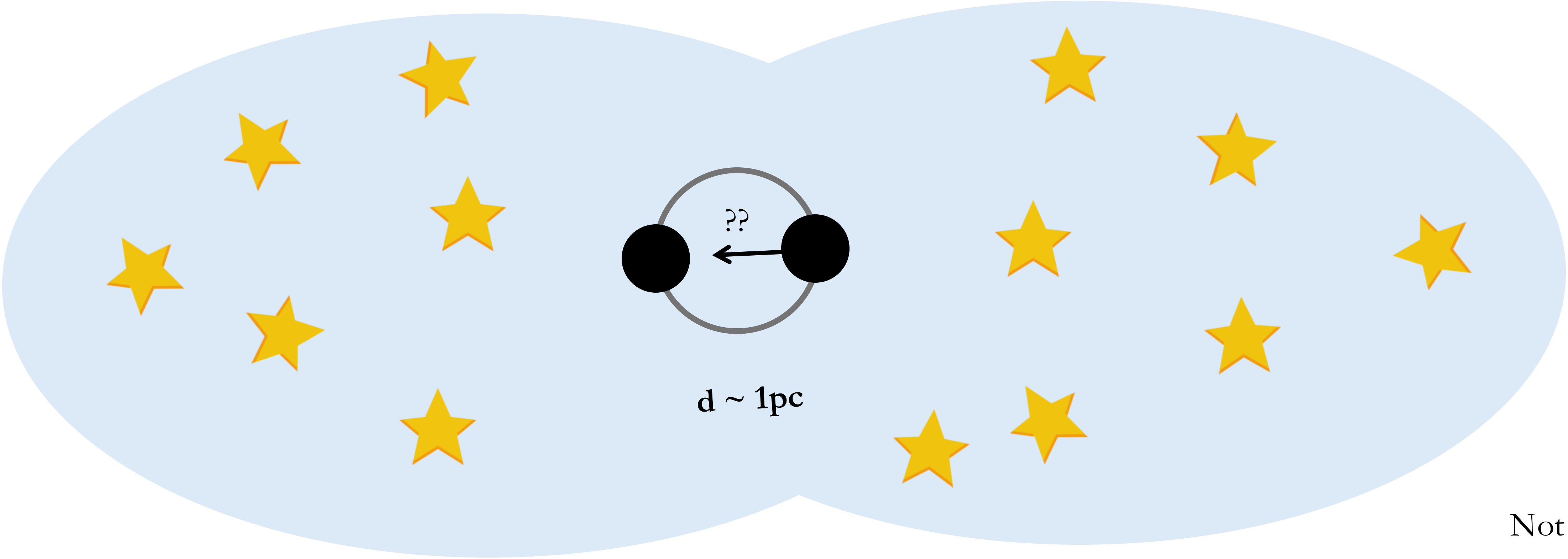
Galaxy mergers are good at getting the BHs to d ~ 1pc



Not to scale

Milosavelic and Merrit 2003
Kazantzidis *et al* 2005 *ApJ* **623** L67

How do we overcome this final parsec?



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1. Dynamical Friction
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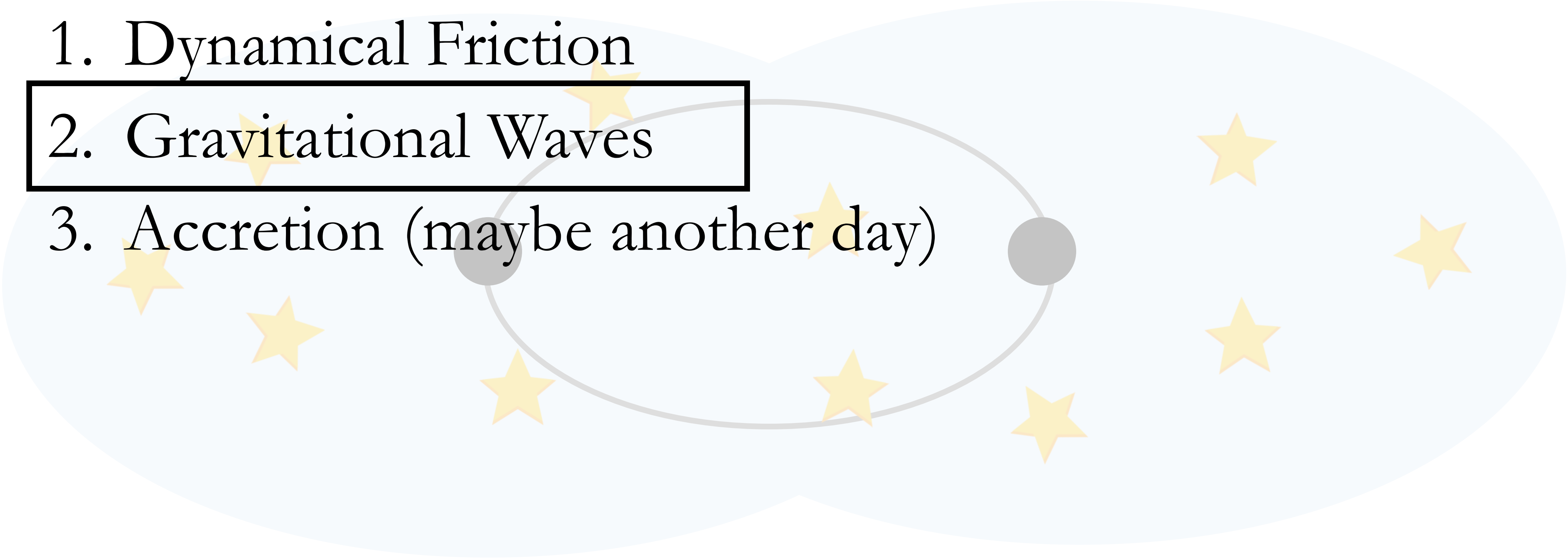
3. Accretion (maybe another day)



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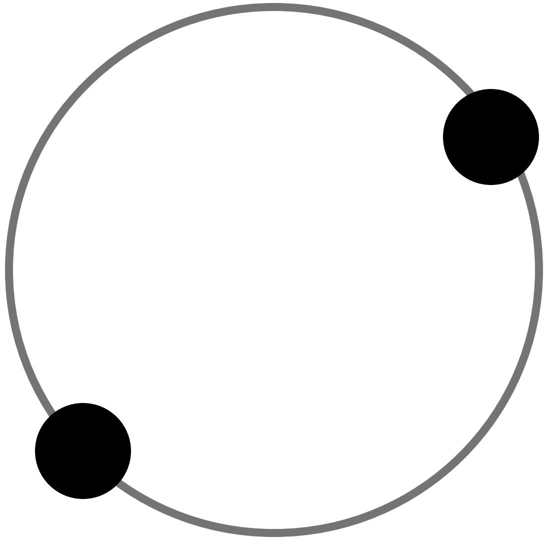
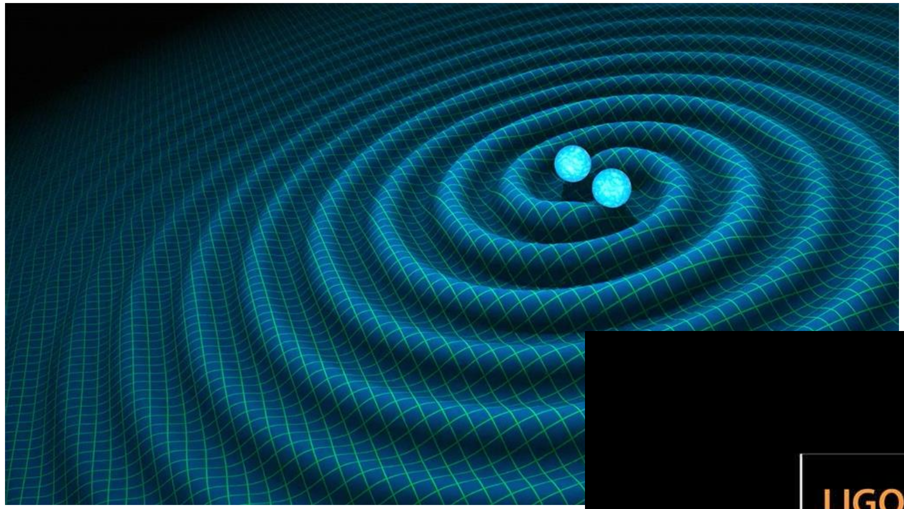
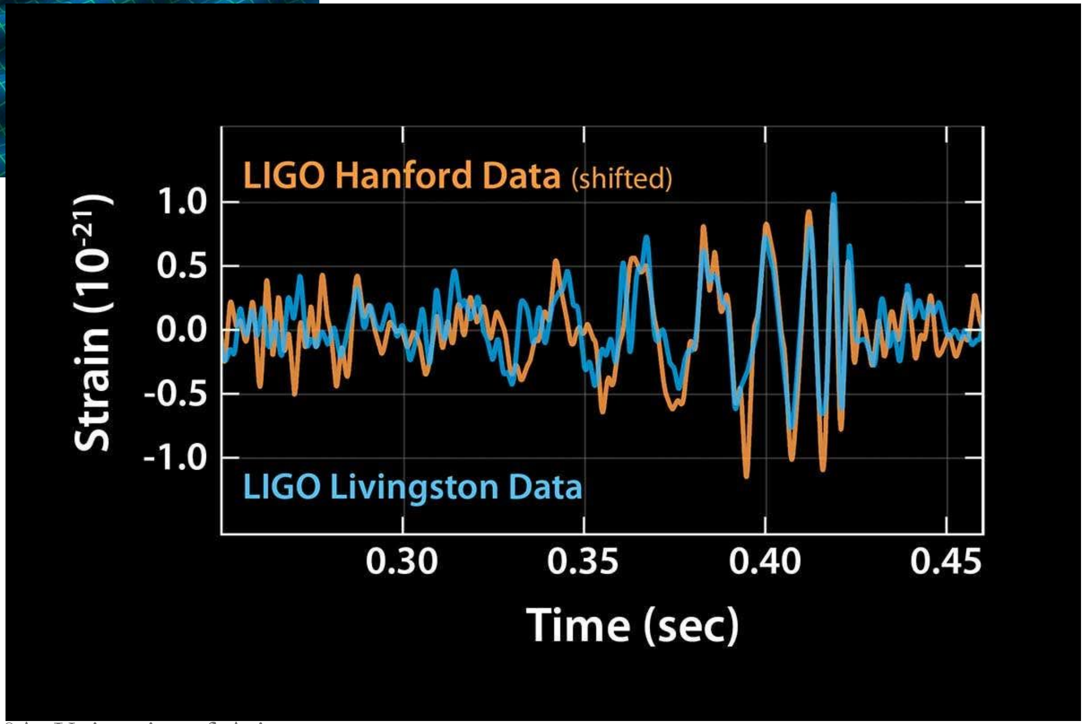
Gravitational Waves (more on Thursday)

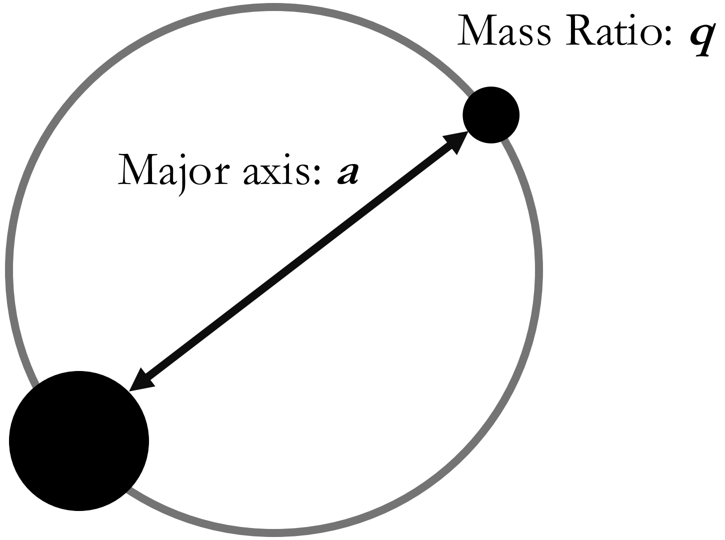






LIGO Collaboration

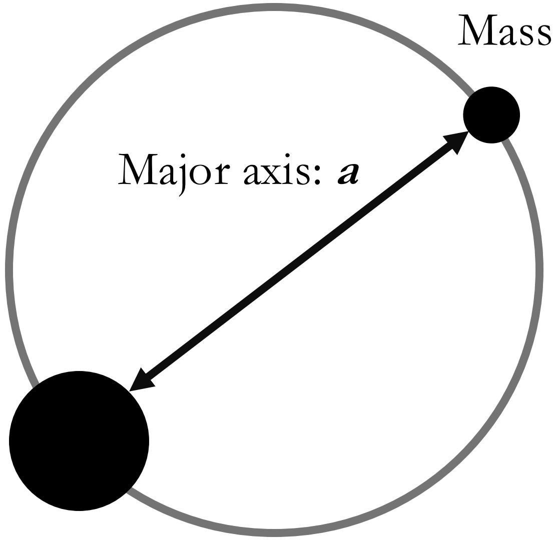
GW Driven Inspiral



Binary Mass: ***M***
Reduced mass: $\mu = M_1 M_2 / (M_1 + M_2)$

- Binary has a total orbital energy
- Gravitational waves emit energy, reducing the binary's total energy
- The binary begins to ***inspiral***

GW Merger Timescale



Binary Mass: ***M***
Reduced mass: $\mu = M_1 M_2 / (M_1 + M_2)$

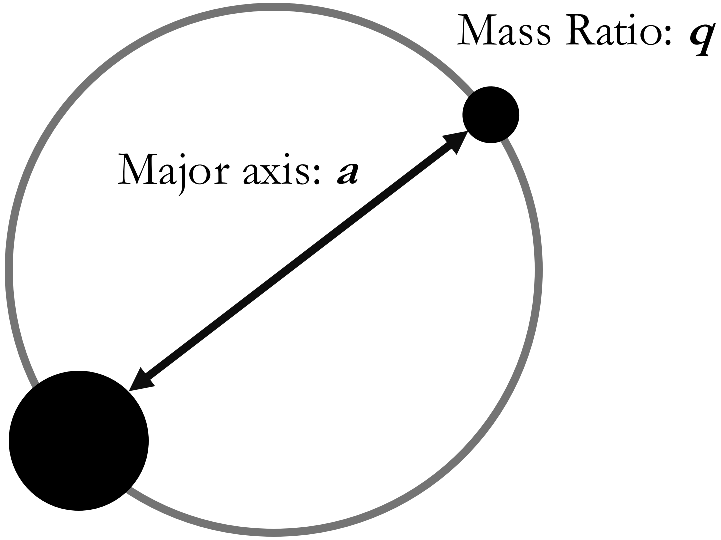
- If the binary has major axis ***a***
- We can define a ***da/dt*** due to GW emission
- Calculate a time scale for **a→0**

Recall HW6 and Dynamical Friction:

$$\frac{d\vec{v}_M}{dt} = -\frac{4\pi \ln(\Lambda) G^2 \rho M}{v_M^2} \hat{v}_M$$

$$\Rightarrow T_{stop} = \frac{v_M^3}{12\pi \ln \Lambda \, G^2 \rho M}$$

GW Merger Timescale (your turn)



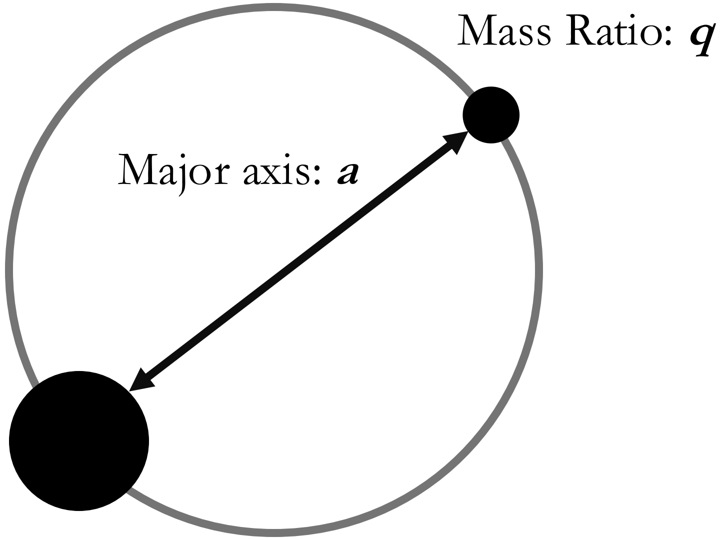
Binary Mass: ***M***
Reduced mass: $\mu = M_1 M_2 / (M_1+M_2)$

- If the binary has major axis ***a***
- We can define a ***da/dt*** due to GW emission
- Your turn: Calculate a time scale for **a→0**
- When integrating, don't forget to define limits

$$\frac{da}{dt} = -\frac{1}{a^3}\frac{64}{5}\frac{G^3 \mu M_1 M_2}{c^5}$$

$$\Rightarrow t_0 = ??$$

GW Merger Timescale



Binary Mass: ***M***
Reduced mass: $\mu = M_1 M_2 / (M_1 + M_2)$

$$\frac{da}{dt} = -\frac{1}{a^3}\frac{64}{5}\frac{G^3 \mu M_1 M_2}{c^5}$$

$$a^3 da = -\frac{64}{5}\frac{G^3 \mu M_1 M_2}{c^5}\, dt$$

$$\int_{a_0}^{0} a^3 da = -\int_0^{t_0} A\, dt$$

$$-\frac{1}{4}a_0^4 = -At_0 \Rightarrow t_0 = -\frac{1}{4A}a^4$$

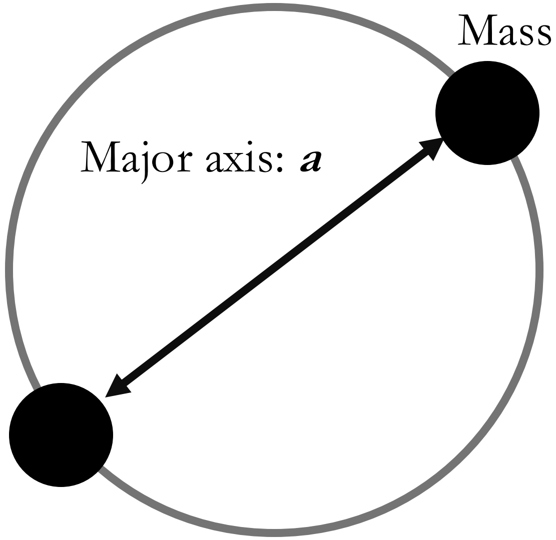
$$t_0 = \frac{5}{256}\frac{c^5}{\mu M_1 M_2 G^3} a_0^4$$

GW Merger Timescale

$$t_0 = \frac{5}{256}\frac{c^5}{\mu M_1 M_2 G^3}a_0^4$$

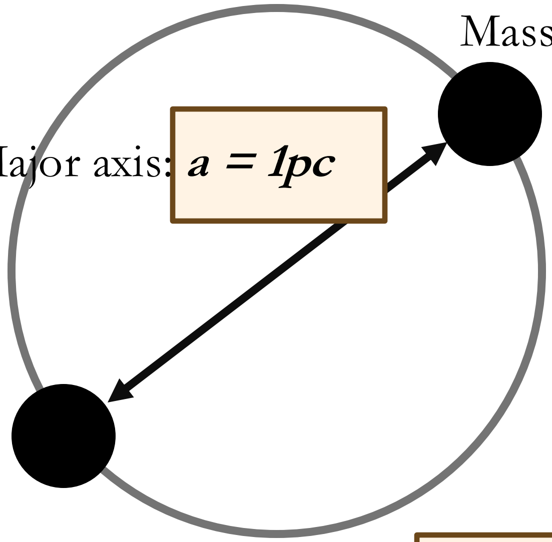
$$t_0 = \frac{5}{16}\frac{c^5}{G^3}\frac{a_0^4}{M^3}$$

$$t_0 = \left(10^{31}\frac{M_{\odot}^3\,Gyr}{pc^4}\right)\frac{a_0^4}{M^3}$$



Binary Mass: ***$M = M_1 + M_2$***
Reduced mass: ***$\mu = M_1M_2/(M_1+M_2) = M/4$***

Calculate the merger time for this binary



$$t_0 = \frac{5}{16}\frac{c^5}{G^3}\frac{a_0^4}{M^3}$$

$$t_0 \sim \left(10^{31}\frac{M_\odot^3\,Gyr}{pc^4}\right)\frac{a_0^4}{M^3}$$

Plugging in the numbers to the left…

$$t_0 \sim \left(10^{31}\frac{M_\odot^3\,Gyr}{pc^4}\right)\frac{(1pc)^4}{\left(10^7 M_\odot\right)^3}$$

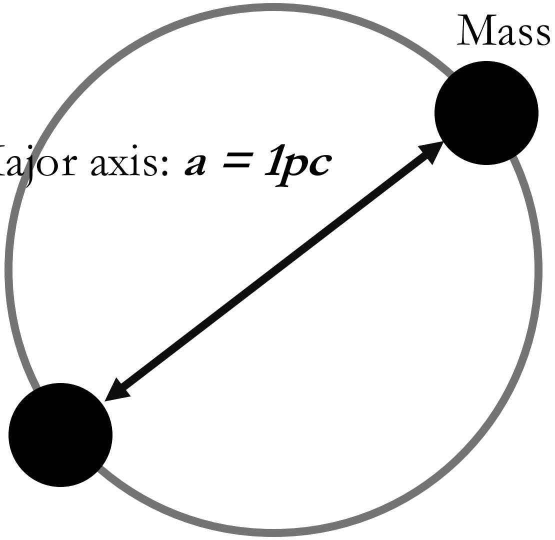
$$t_0 \sim 10^{10}\,Gyr$$

The universe is only ~14 Gyr old!!

$$t_0 \sim 10^{10} Gyr$$

Mass Ratio: ***q = 1***

Major axis: ***a = 1pc***

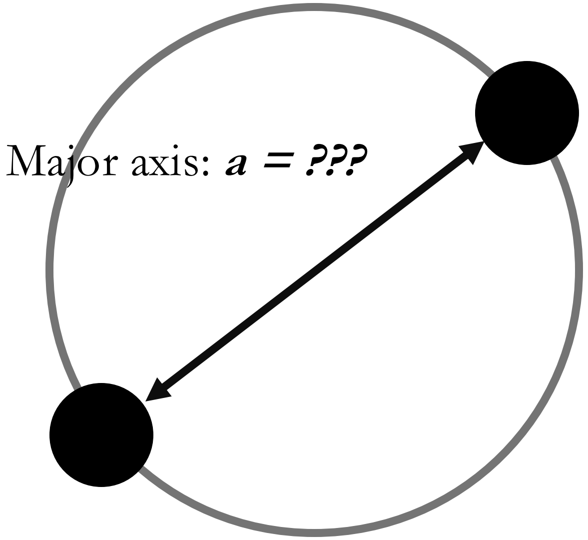


Binary Mass: $\boldsymbol{M = M_1 + M_2 = 10^7}$ ***Msun***

Reduced mass: $\boldsymbol{\mu}$

Gravitational waves would take ***too long*** to drive the binary to merger

At what separation are GWs efficient?



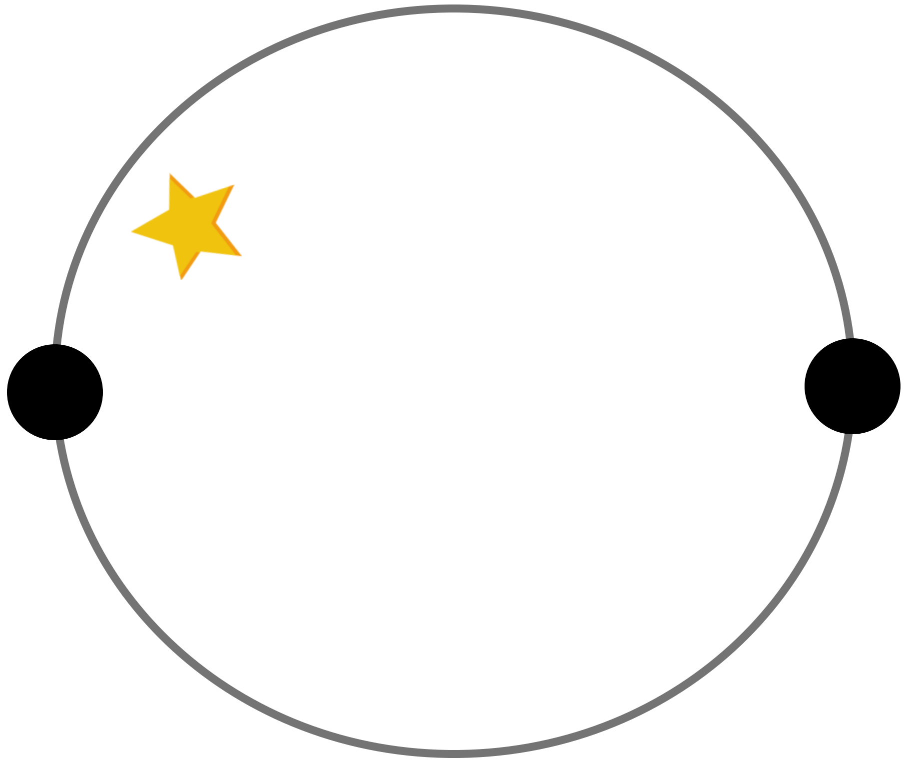
$$t_0 \sim \left(10^{31}\,\frac{M_\odot^3\,Gyr}{pc^4}\right)\frac{a_0^4}{M^3}$$

$$a_0^4 \sim \left(10^{-31}\,\frac{pc^4}{M_\odot^3\,Gyr}\right)t_0 M^3$$

Let t_0***=10Gyr*** (Hubble time):

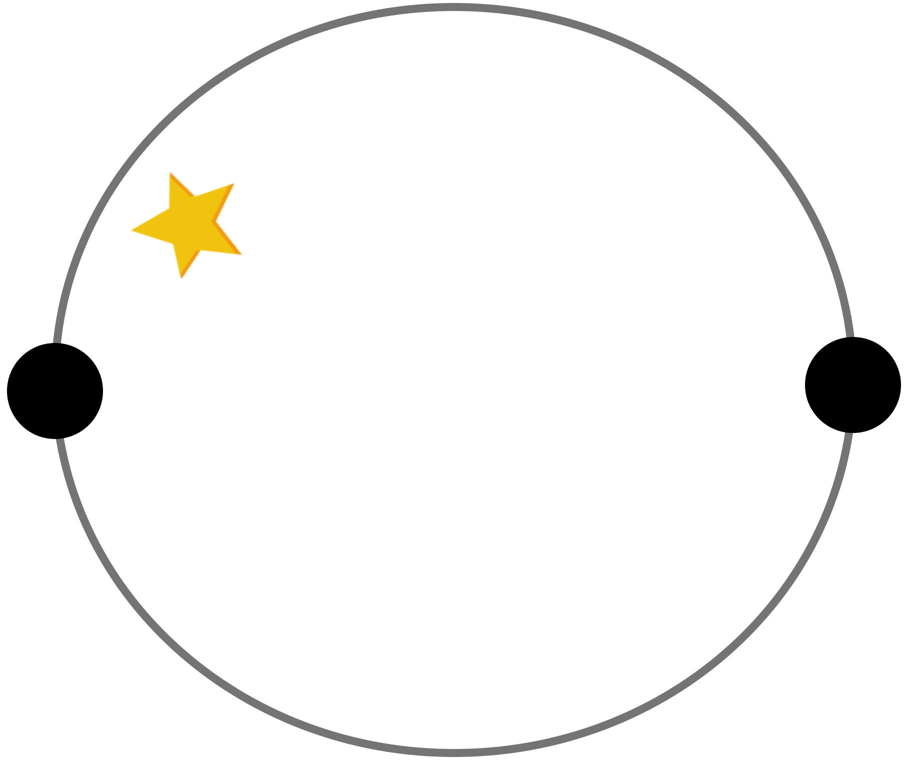
$$a_{GW} \sim 5\,mpc\left(\frac{M}{10^7 M_\odot}\right)^{3/4}$$

We have run into the ‘final parsec problem’!



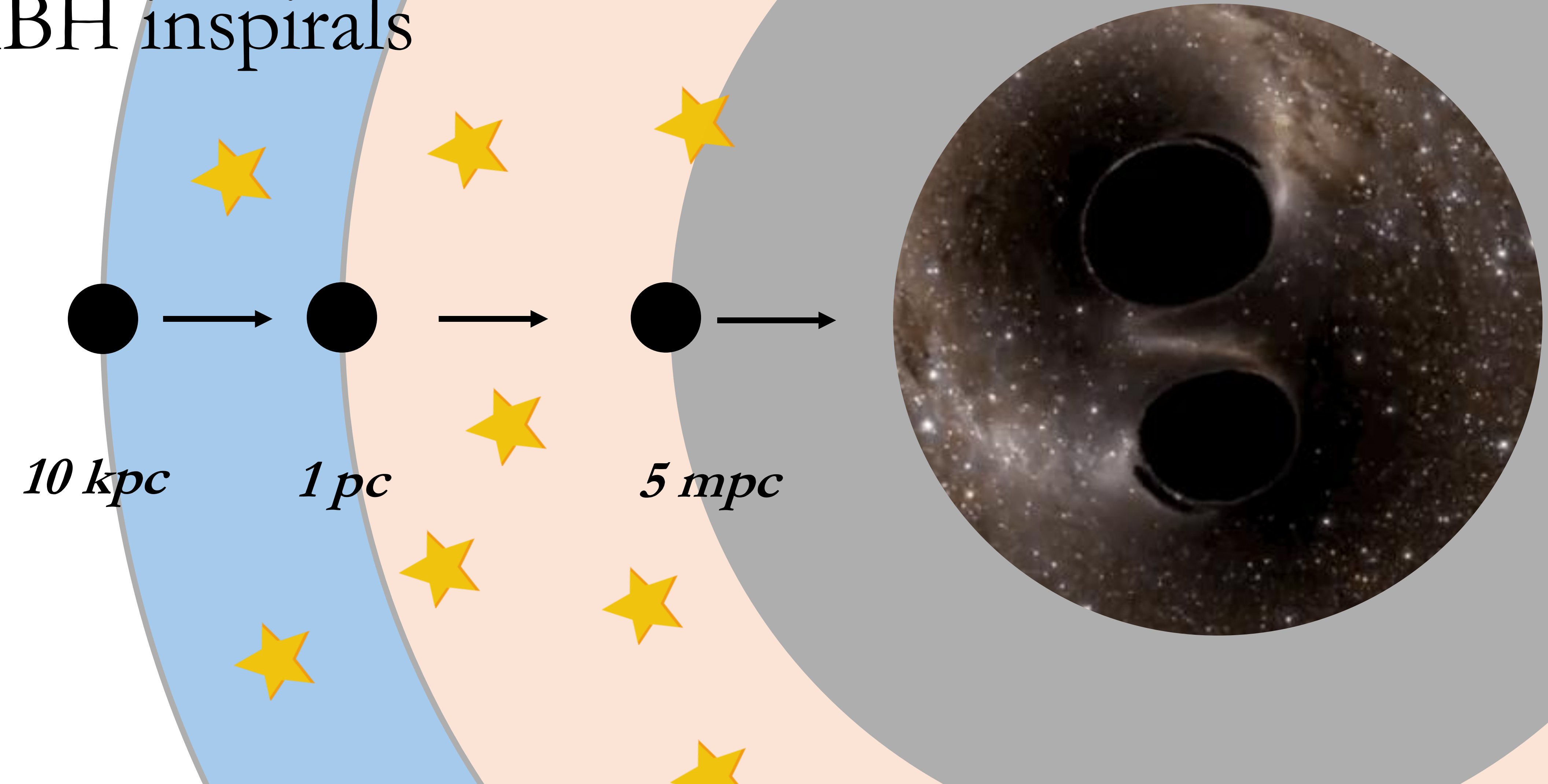
- Not enough stars in the BHs ‘loss cone’
- Therefore, not enough stellar ejections and the SMBHs ‘stall’ at this radius
- This is the very well known ‘final parsec problem’
- Milosavljevic & Merritt 2003

The ‘final parsec problem’ is solved!



- Weird galaxy shapes allow the loss cone to be refilled by different stellar orbits (Gualandris et al. 2016)
- There are enough stars for the SMBHs to eject and get closer together
- Different types of dark matter can even solve this problem (Alonso-Alvarez et al. 2024.)

Multiple stages and processes to SMBH inspirals



On Thursday

- Don’t forget to submit your notes for the in-class assignment?
- Rest of today’s lecture
- More about my research! What is the cutting-edge research today?
- Playing with gravitational waveforms – notebooks and open-source science